Beyond Carbon Pricing: Integrating Mitigation, Adaptation, and Carbon Removal*

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Abstract

Relying solely on carbon pricing to meet Paris Agreement targets imposes prohibitive economic costs. We show that achieving the 2°C stabilization goal through taxation alone requires carbon prices reaching approximately $474/tCO_2$, a level that triggers widespread capital divestment. To resolve this dilemma, we develop a dynamic stochastic integrated assessment model that optimizes a portfolio of carbon taxation, clean-capital subsidies, adaptation investment, and carbon dioxide removal (CDR). Our analysis identifies CDR as a necessary condition for stabilization rather than a supplementary measure. In the optimal portfolio, net carbon removal scales from 0.04 to 3.7 $GtCO_2$/year by 2050 to maintain temperature targets at a feasible cost. These instruments act as economic complements: carbon pricing and subsidies target new emissions, CDR reduces the legacy atmospheric stock, and adaptation protects the economic base from immediate damages. Consequently, the welfare gains from the integrated portfolio significantly exceed the sum of individual instrument effects. We conclude that optimal climate policy requires shifting from a price-centric framework to a diversified approach in which carbon removal and adaptation serve as core pillars of decarbonization.

JEL classification: Q54, Q58, H23, D62, D81, O44
Keywords: climate policy mix; carbon tax; clean subsidies; adaptation; carbon dioxide removal; integrated assessment model; social cost of carbon; disaster risk

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1 Introduction

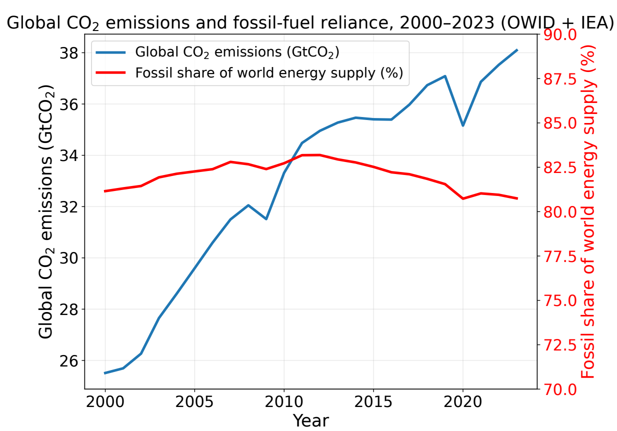
The Paris Agreement commits signatories to hold the increase in global mean temperature to "well below" 2°C above pre-industrial levels and to pursue efforts to limit warming to 1.5°C.[1] A large and growing scenario-assessment literature concludes that currently implemented policies remain inconsistent with these temperature limits. For example, the Climate Action Tracker (CAT) estimates end-of-century warming of about 2.7°C under current policies, while full achievement of stated pledges and targets would reduce this to roughly 2.1°C—still above the Paris benchmark.[2] Hence, the policy gap is not primarily about the existence of long-run targets; it is about the pace and composition of near-term action required to make those targets credible.

Figure 1 documents four facts that illustrate this conclusion. First, Panel a) shows that global CO_2 emissions have grown steadily over the past two decades, interrupted only by the COVID-19 pandemic in 2020. Fossil fuels continue to supply roughly 80% of global energy. Second, Panel b) plots observed temperature anomalies, which confirm ongoing warming; indeed, the rate of surface temperature increase has accelerated in recent years. Third, Panel c) displays the emission reductions required to limit warming to 1.5°C or 2°C. The necessary cuts are large and must begin soon. Fourth, Panel d) shows that the remaining carbon budgets implied by these targets are small relative to current emission flows, leaving little time for gradual adjustment.

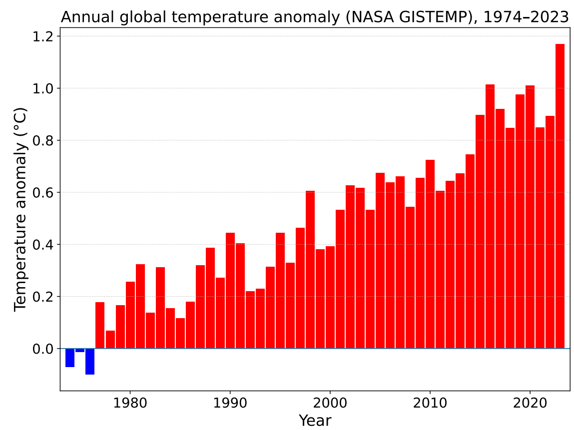
These patterns are consistent with the broader empirical evidence on the ongoing energy transition. Improvements in energy intensity have slowed relative to the pace required for rapid decarbonization, and although renewable deployment has accelerated, the necessary scale-up remains substantial.[3] Baseline projections continue to imply persistent oil and gas demand and a fossil supply structure dominated by established producers through mid-century.[4]

Against this background, the central normative benchmark in climate economics remains clear: under standard assumptions, a Pigouvian carbon tax that prices the social cost of carbon is the efficient instrument for mitigating the externality[5]. If the carbon price is correctly set and credibly implemented, private incentives can, in principle, deliver an efficient allocation of abatement across technologies and sectors. Yet two considerations motivate a broader

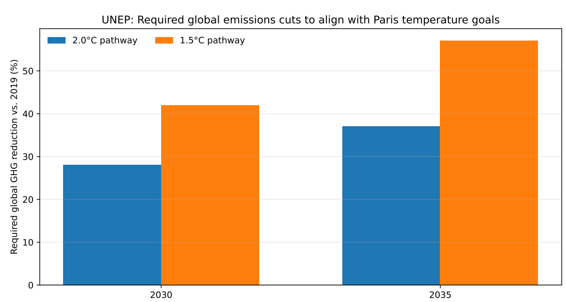
[1] See (United Nations Framework Convention on Climate Change, 2015).
[2] See (Climate Action Tracker, 2021).
[3] See International Energy Agency (2024a) and International Energy Agency (2024b).
[4] See International Energy Agency (2025).
[5] Standard references related to the pricing of carbon are (Nordhaus, 1993; Golosov et al., 2014a) among others



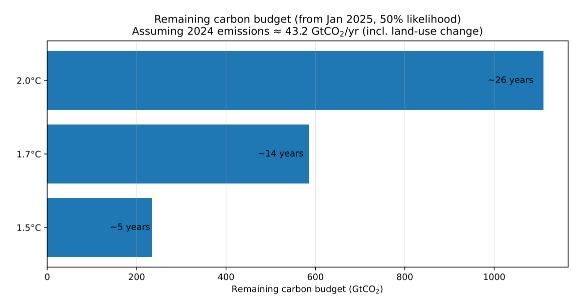
(a) CO_2 emissions and fossil reliance (2000–2023).



(b) Observed warming signal (last 50 years).



(c) Required global cuts under 2°C vs. 1.5°C benchmarks.



(d) Remaining carbon budget expressed as "years left" at current emissions.

Figure 1: Progress indicators and Paris-consistent benchmarks. Panel (a) documents persistent emissions and fossil dependence; panel (b) shows the realized warming signal; panels (c)–(d) translate temperature targets into near-term mitigation requirements and a limited remaining carbon budget.

approach.

First, political economy and transition constraints frequently bind, limiting both the level and the time profile of feasible carbon prices. The distribution of near-term losses is concentrated (e.g., in fossil-intensive industries, asset owners, and regions), while gains are diffuse and delayed; credibility of revenue recycling and compensation is often limited; and concerns about abrupt capital scrapping and macroeconomic disruption can dominate policy choice. When such constraints bind, the relevant policy problem is second-best: not "set the Pigouvian tax," but design an implementable *portfolio* of instruments with high environmental efficacy and manageable transition costs.

Second, the climate problem is inherently a *stock* problem. Atmospheric CO_2 persists for centuries, and the climate system exhibits substantial thermal inertia, implying that reductions in the flow of new emissions do not quickly unwind the existing atmospheric stock. While the "zero-emissions commitment" literature emphasizes that warming stabilizes after net CO_2 emissions reach (approximately) zero, the accumulated stock prevents a rapid decline in temperatures absent active removal. A pure flow instrument can therefore require progressively higher marginal incentives as the legacy stock grows. This stock dimension becomes more salient when macroeconomic damages exceed those embedded in early integrated assessment models, as suggested by recent work that exploits temperature variation and long-run responses.

Our paper aims to address these issues. In particular, we develop a stochastic integrated assessment model in which the first-best carbon tax is not viable as it would lead to substantial divestment of the dirty capital stock, and in turn, since the dirty capital stock is currently larger than the clean capital stock, would lead to substantial output losses if the optimal carbon tax were fully implemented. Instead, our optimal policy proposal is a multi-instrument approach that acknowledges that, for the foreseeable future, the dirty capital stock will, due to the political economy considerations mentioned above, continue to play an essential role in global energy supply; thus, CO_2 emissions will take longer to decline.

To this end, we develop a framework in which optimal climate policy consists of a portfolio of policies. More precisely, we jointly optimize (i) a carbon tax that prices current emissions, (ii) clean-capital subsidies and directed innovation policy that accelerate the transition away from fossil-based production, (iii) adaptation investment that reduces the elasticity of economic damages to temperature and climate risk, and (iv) carbon dioxide removal (CDR) that actively draws down the atmospheric stock. The key implication is that, under implementability constraints and stock dynamics, a policy mix can dominate a carbon-price-only approach in both feasibility and welfare terms.

To quantify this prior claim and discipline our model, we calibrate it to the RCP4.5 pathway because it provides a policy-relevant intermediate benchmark—neither business-as-usual nor an extreme near-term decarbonization scenario—under which stabilization requires substantial but gradual adjustment, consistent with capital inertia. This is precisely the environment where a multi-instrument framework is most informative: meeting RCP4.5-type trajectories typically cannot be achieved by carbon pricing alone without shifting adjustment to constrained margins, and instead calls for a portfolio that combines mitigation, clean reallocation, resilience, and atmospheric stock management, including carbon dioxide removal to offset residual emissions.

Our analysis yields four main results. First, no single instrument suffices. Even combining carbon taxes with clean subsidies cannot achieve the required emissions path without CDR, because these instruments address only the flow of new emissions while the existing atmospheric stock locks in current temperature levels. The tax rate required to meet the 2 °C target through pricing alone would force widespread capital divestment and impose prohibitive economic costs.

Second, carbon dioxide removal is essential, not optional. In our calibrated model, achieving the 2 °C target requires scaling carbon capture from current levels of approximately 0.04 $GtCO_2$ per year to 3.7 $GtCO_2$ per year by 2050, eventually reaching 6.5 $GtCO_2$ per year by 2100. This finding challenges the view that negative emission technologies are a backstop for the distant future. In our framework, they are a necessary component of the medium-term transition, constrained only by the prudent planetary storage limit of approximately 1,460 $GtCO_2$.

Third, the optimal policy mix evolves over time. In the near term, from 2020 to 2035, the portfolio relies heavily on clean capital subsidies to overcome path dependence in energy infrastructure, complemented by moderate carbon taxes. As the clean transition matures, the portfolio shifts. Adaptation spending rises gradually with temperatures and disaster frequency. Carbon capture accelerates sharply after 2030 as the imperative shifts from reducing flows to drawing down the stock. By 2050, CDR will become decisive. This dynamic pattern reflects the distinct timescales on which different market failures operate: mitigation addresses the flow problem, adaptation manages ongoing damages, and removal tackles the legacy stock.

Fourth, the optimal policy mix is state - and parameter-dependent. Because the four instruments operate on distinct margins—reducing emissions flows, accelerating clean capital formation and innovation, lowering damage exposure through adaptation, and managing the atmospheric carbon stock through removal—their relative importance depends on the underlying economic and climate environment. As a result, changes in key structural features of the model (such as the severity of climate tail risk, the effectiveness of adaptation, or the strength of natural carbon uptake) reallocate the optimal response across instruments rather than simply scaling all policies up or down. This property is central for interpretation: it allows the model to translate uncertainty about climate damages and transition constraints into disciplined predictions about how the composition of the policy portfolio should adjust.

To assess the robustness of our quantitative conclusions, we conduct a structured sensitivity analysis that perturbs a set of economically meaningful drivers of climate policy. The resulting comparative statics are informative about both mechanism and magnitude. Carbon pricing is most sensitive to the tail-risk component of climate damages: when disaster risk is higher, the optimal carbon tax rises markedly; when it is lower, the carbon tax falls correspondingly. In contrast, variation in natural carbon uptake and adaptation effectiveness has a more muted impact on the carbon tax itself, though these parameters matter strongly for the evolution of climate outcomes and the damage burden. Consistent with these channels, the implied temperature path responds materially to shifts in discounting and tail risk, highlighting that uncertainty about catastrophic climate risk and intertemporal trade-offs can materially alter optimal mitigation stringency even when the broader policy architecture remains unchanged.

The same sensitivity analysis clarifies the determinants of aggregate economic damages. Expected damages respond most strongly to adaptation effectiveness and to disaster risk, reflecting both a direct channel—adaptation reduces the damage intensity conditional on warming—and an indirect channel—changes in the damage environment feed back into mitigation incentives and thereby into the temperature trajectory. Quantitatively, plausible variation in adaptation effectiveness generates large swings in the cumulative damage burden over the medium run, underscoring that adaptation is a central determinant of the macroeconomic cost of climate change.

We also examine the role of carbon removal policy and the extent to which stock management is required to achieve stabilization. In the calibrated benchmark, optimal carbon removal scales from current levels near 0.04 $GtCO_2$/year to about 3.7 $GtCO_2$/year by 2050 and peaks around 6.5 $GtCO_2$/year by 2100, indicating that carbon removal is quantitatively essential well before the end of the century. Increasing carbon-removal effectiveness strengthens the long-run temperature response, while tightening the implicit concentration objective requires substantially more aggressive removal. These results reinforce our paper's central mechanism: because climate stabilization is a stock problem, policies that directly manage the atmospheric stock are natural complements to flow-based mitigation.

Finally, we present a policy counterfactual that illustrates why a portfolio approach is important. If carbon capture is shut down, meeting the 2°C limit requires a carbon tax of about \$474/tCO_2 (first best), versus about \$178/tCO_2 in the benchmark; even the implementable tax rises from about \$51 to \$214/tCO_2. Conversely, if the carbon tax is removed, meeting the same temperature objective requires carbon removal exceeding roughly 25.45 GtCO_2/year by mid-century, compared with about 3.77 GtCO_2/year in the benchmark. At plausible unit costs (\$100–250/tCO_2), that scale implies annual expenditures of roughly \$2.4–6 trillion (several percent of global GDP), and it runs into a hard physical constraint: sustaining removal near 24 GtCO_2/year would exhaust a prudent planetary storage limit of 1,460 GtCO_2 within roughly six decades. The implication is that single-instrument stabilization pushes adjustment onto a constrained margin—either extreme carbon pricing and accelerated fossil-capital contraction, or implausibly large removal—whereas a coordinated policy mix achieves stabilization with economically and physically more plausible reliance on each instrument.

The remainder of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 develops the integrated assessment model, incorporating climate dynamics through an enhanced two-component energy balance model and specifying the economic framework with clean and dirty capital stocks. Section 4 derives the closed-form solution for the policy portfolio. Section 5 analyzes how these instruments affect capital accumulation and emissions trajectories. Section 6 presents the calibration strategy. Section 7 examines sensitivity and demonstrates that achieving the 2 °C target requires the full portfolio. Section 8 concludes.

2 Related literature

The canonical benchmark in climate economics is that efficient mitigation can be implemented through a Pigouvian carbon price that internalizes the social cost of emissions, as in the integrated assessment tradition pioneered by Nordhaus (1993) and formalized in general equilibrium by Golosov et al. (2014a). In this view, a credibly implemented carbon price can decentralize an efficient allocation of abatement effort across sectors and technologies. A parallel body of literature debates the magnitude and persistence of macroeconomic damages from global warming. Empirical work documenting nonlinear temperature–output relationships implies that damages can rise rapidly at higher temperatures, as shown in (Burke et al., 2015), and recent macro evidence emphasizes potentially significant long-run effects of temperature changes as documented in (Bilal and Känzig, 2024). These contributions sharpen the quantitative stakes of mitigation, but they do not resolve an important design tension emphasized in our paper: carbon pricing primarily targets the *flow* of new emissions, while climate stabilization is ultimately governed by the *stock* of atmospheric carbon and the inertia of the climate system.

This flow–stock distinction is central in climate science and in simple climate modeling. Atmospheric CO_2 remains elevated for long periods, and warming does not quickly reverse absent active stock management (Solomon et al., 2009; Archer et al., 2009; Joos et al., 2013b). The assessment literature further highlights that temperature dynamics reflect substantial inertia and adjustment delays even after emissions decline (IPCC, 2021b). These physical features motivate modeling approaches that combine tractability with interpretable dynamics, including reduced-form energy-balance representations and parsimonious carbon-cycle modules (Gregory, 2000; Winton et al., 2010; Glotter et al., 2014). Our framework builds on this line by using a compact climate block suitable for policy optimization while still capturing persistence, inertia, and changing sink behavior that matter for welfare-based policy design.

A second motivation for moving beyond carbon pricing alone comes from the political economy and transition-management literature. A large carbon price can create concentrated, salient near-term losses for fossil-intensive firms, asset owners, and regions, while benefits are diffuse and arrive with delay; limited credibility of revenue recycling and compensation can further depress acceptability. This perspective shifts the practical problem from choosing the Pigouvian tax to designing implementable policy packages that achieve substantial mitigation with manageable transition costs.[6] This "policy packaging" logic directly motivates portfolio approaches that combine pricing with complementary instruments operating on other margins, thereby broadening political support.

A closely related literature explains why technology policy is a natural complement to carbon pricing. With innovation externalities and path dependence, subsidies and directed innovation policies can accelerate clean technological change beyond what carbon pricing alone would induce, especially when prices are constrained or delayed as in (Acemoglu et al., 2012b; Popp, 2010; Aghion et al., 2016). A further strand endogenizes adaptation within integrated assessment frameworks, emphasizing that resilience investment can reduce damage exposure and can be especially valuable when mitigation is imperfect or delayed as discussed in (Hallegatte, 2009; De Bruin et al., 2009; Dietz and Stern, 2015). Closely related to our work is the paper by Hong et al. (2023), in which they develop a continuous-time stochastic general-equilibrium model with climate-related disasters in which optimal adaptation—a mix

[6] See for instance(Jenkins, 2014; Klenert et al., 2018; Carattini et al., 2018; Meckling et al., 2015).

of private and public mitigation of disaster losses to capital—is financed alongside carbon pricing via capital taxation; learning about the climate–disaster linkage shapes both the social cost of carbon and asset prices. In our setting, these channels are not add-ons: they are quantitatively relevant margins that interact with mitigation and shape the time profile of optimal policy.

Finally, a rapidly expanding climate-policy literature emphasizes the role of carbon dioxide removal (CDR) and negative emissions as a mechanism for managing the atmospheric stock when residual emissions persist. Reviews synthesize the technology landscape and the scale challenges of negative emissions include the work by (Minx et al., 2018; Fuss et al., 2018), and the paper by (Realmonte et al., 2019) discusses the role of specific removal technologies such as direct air capture in deep mitigation pathways. Assessments also stress that CDR deployment is constrained by feasibility, resources, and storage considerations.[7] Complementing this, climate-finance and uncertainty-sensitive valuation work highlights the importance of tail risk and model uncertainty for social valuation and optimal policy as discussed in (Weitzman, 2011; Pindyck, 2013b; Barnett et al., 2019; Mittnik et al., 2019). Our contribution is to integrate these strands—including the disaster–adaptation perspective of Hong et al. (2023)—into a unified stochastic general equilibrium IAM in which mitigation, clean subsidies/innovation, adaptation, and CDR are jointly chosen, allowing the model to deliver disciplined predictions not only about the *level* of climate action but also about the *composition* and evolution of an implementable optimal policy portfolio.

3 Equilibrium Framework

We now lay out the model. The framework is a stochastic equilibrium integrated assessment model that couples a tractable climate module with a dynamic macroeconomic environment, so that emissions, atmospheric carbon, temperature, and economic decisions jointly evolve over time. The climate block formalizes how human and natural emissions accumulate in the atmosphere, how carbon uptake changes endogenously with warming, and how nonlinearities can arise through abrupt emissions spikes. Building on these climate dynamics, the economic block describes production and capital accumulation with clean and dirty technologies, endogenous clean productivity growth through R&D, and climate damages.

[7] See (IPCC, 2022; Gidden et al., 2025).

3.1 Modeling the climate

Climate dynamics play a fundamental role in shaping the economic outcomes of mitigation and adaptation policies. Understanding how emissions accumulate in the atmosphere, how natural carbon sinks respond over time, and how climate tipping points amplify risks is essential for designing optimal economic interventions. In our framework, we explicitly model the evolution of atmospheric CO_2 and temperature using a two-box energy balance system, capturing both anthropogenic and natural influences on the climate. This representation allows us to quantify how different emissions pathways alter future climate states and, in turn, affect economic decisions regarding mitigation investments, adaptation expenditures, and technological innovation.

3.1.1 The atmospheric concentration of CO_2 and emissions

We denote the atmospheric concentration of CO_2 by $E_{S,t}$. The evolution of $E_{S,t}$ is driven by three distinct sources of emissions: (i) human-made emissions, $E_{H,t}$, which depend on investment in polluting (dirty) capital and the level of investment into carbon capture (or negative emissions) technologies, (ii) natural emissions, $E_{N,t}$, which occur exogenously and remain largely beyond human control, and (iii) feedback effects, which emerge as climate tipping points amplify emissions in response to human-induced changes in the environment. The accumulation of atmospheric CO_2 follows the dynamic process

$$\begin{aligned} dE_{S,t} &= \zeta_E \left(dE_{H,t} + dE_{N,t}\right) - \mathcal{H}(E_{S,t}, CF_t)dt \\ &= \zeta_E \left[g_H(I_{dt}, G_{cc,t})dt + \bar{E}_N dt + Z_{FB} dN_t^{\lambda_{E_S}}\right] - \mathcal{H}(E_{S,t}, CF_t)dt,\ Z_{FB} \sim F_{Z_{FB}}(\cdot), \end{aligned} \tag{1}$$

where ζ_E is a scaling factor that converts emissions (measured in tons of CO_2) into changes in atmospheric concentration (measured in ppm), $g_H(I_{dt}, G_{cc,t})$ captures the net human-made emissions as a function of investment in dirty capital I_{dt} and investment in carbon capture technologies $G_{cc,t}$ (both formally introduced in Section 3.2), $\bar{E}_N$ is the constant rate of natural emissions, and $\mathcal{H}(E_{S,t}, CF_t)$ denotes the carbon sink function that depends on the level of CO_2 in the atmosphere $E_{S,t}$ as well as a climate-feedback process CF_t (defined below in Section 3.1.2). The term $Z_{FB} dN_t^{\lambda_{E_S}}$ represents jump risk: $N_t^{\lambda_{E_S}}$ is a Poisson counting process with intensity λ_{E_S}, and $Z_{FB} \sim F_{Z_{FB}}(\cdot)$ is the random jump size, capturing the possibility of abrupt increases in emissions due to climate-induced shifts such as permafrost thawing or large-scale forest dieback.[8] We parametrize the net human-impact function as

$$g_H\big(I_{dt}, G_{cc,t}\big) \;=\; \eta_d u_t(\tau_{d,t}) K_{d,t} - f_{cc}\big(G_{cc,t}; E_{S,t}, \bar{E}_T, \nu_{cc}, \xi\big),$$

where $\eta_d > 0$ is the emissions intensity of dirty capital (tons of CO_2 per unit of capital), $K_{d,t}$ is the stock of dirty capital, and $f_{cc}(\cdot) \geqslant 0$ represents the negative emissions technology function, which is increasing and concave in carbon capture investment $G_{cc,t}$. The utilization function $u_t(\tau_{d,t}) = \underline{u} \;+\; (1-\underline{u})\exp\Big(-\rho_u \frac{\tau_{d,t}}{r_d}\Big)$, with $\rho_u \geqslant 0$, captures how the carbon tax $\tau_{d,t}$ reduces emissions at the intensive margin through reduced utilization of existing dirty capital. Here, $r_d = \nu_d FF - \delta_d$ denotes the net (of depreciation δ_d) return on dirty capital investment, where ν_d is the productivity parameter of dirty capital and FF represents fossil fuel inputs (both formally defined in Section 3.2). The constraint $\tau_{d,t} < r_d$ ensures that dirty capital maintains positive post-tax returns; if the carbon tax is at or above r_d, firms would cease all dirty capital operations, rendering the utilization function undefined.

We incorporate this utilization channel so that the carbon tax, apart from affecting investment into dirty capital stock, also directly reduces emissions. This utilization channel serves as an immediate cost pass-through mechanism for the carbon tax, as firms typically respond quickly by reducing operating hours or switching to cleaner inputs, rather than waiting years to tear down and rebuild plants.[9]

3.1.2 Enhanced two-component EBM with time-varying carbon uptake

Our framework extends the classical two-component energy balance model (EBM) by incorporating time-varying carbon uptake and heat capacity, recognizing that nature's ability to absorb CO_2 evolves over time.[10] Combining the emission sources from Section 3.1.1 with

[8] The mathematical structure aligns with literature on carbon cycle dynamics and tipping points as in Lenton et al. (2008); Cai et al. (2016), where nonlinearities and feedback loops play a crucial role in determining future climate outcomes.

[9] Moreover, our specification $dE_{H,t} = \eta_d u_t(\tau_{d,t}) K_{dt} dt$ is exactly the intensive-margin analogue of the abatement policy μ_t in the DICE model, i.e. in DICE, emissions are $E_t = \sigma_t(1-\mu_t)Y_t$, where σ_t is emissions intensity, Y_t is output, and $\mu_t \in [0,1]$ is the abatement rate. Abatement entails a convex resource cost $\Psi(\mu_t) = \frac{\theta}{2}\mu_t^2 Y_t$, so the planner (or a decentralized firm facing a carbon price) chooses μ_t to minimize $\Psi(\mu_t) + \tau_t\,\sigma_t\,(1-\mu_t)\,Y_t$, i.e., the sum of abatement cost and the tax paid on residual emissions. The first-order condition is $\theta\mu_t Y_t = \tau_t\sigma_t\, Y_t \implies \mu_t^\star = \dfrac{\tau_t\,\sigma_t}{\theta} \in [0,1]$. Thus, as the abatement efforts in the DICE model, our utilization function u_t is directly affected by the carbon tax.

[10] Our framework shares some features with the well-known DICE (Dynamic Integrated Climate-Economy) model by Nordhaus (1992), but diverges in key aspects of how the climate system is represented. DICE employs a simplified, three-box carbon cycle and a largely linear structure for carbon flow among the atmosphere, upper ocean, and lower ocean layers. While DICE does model temperature with a two-layer approach, it generally does not include a separate, time-varying climate-feedback variable that captures changes in natural carbon sinks over time. Instead, carbon uptake in DICE follows parameterized decay equations that remain largely

natural carbon sinks, the complete evolution of atmospheric CO_2 concentration $E_{S,t}$ is given by

$$dE_{S,t} = \Big\{ \underbrace{\zeta_E \Big(\eta_d u_t(\tau_{d,t}) K_{d,t} + \bar{E}_N \Big)}_{\text{Total new emissions}} - \underbrace{\Big(\delta_{E_S} E_{S,t} + \xi_{CF}\, CF_t \Big)}_{\text{Natural CO}_2\text{ absorption}} - \underbrace{f_{cc}\big(G_{cc,t}; E_{S,t}, \bar{E}_T, \nu_{cc}, \xi\big)}_{\text{CO}_2\text{ removal}} \Big\} dt + \underbrace{\zeta_E Z_{FB}\, dN_t^{\lambda_{E_S}}}_{\text{Tipping Point}}, \tag{2}$$

where the first term captures total new emissions (human-made and natural), the second term represents natural CO_2 absorption by oceans and terrestrial systems, the third term captures anthropogenic carbon removal, and the final term accounts for abrupt emission spikes from climate tipping points. The natural absorption consists of two components: $\delta_{E_S} > 0$ is the baseline natural carbon removal rate (approximately 0.26% per year in our calibration, implying an atmospheric CO_2 lifetime of several centuries), and $\xi_{CF} > 0$ measures how the time-varying climate-feedback process CF_t affects sink capacity. The climate-feedback variable CF_t captures the time-varying impact of temperature rise on natural carbon sinks. Its dynamics are governed by

$$dCF_t = \kappa_{CF}\big(\theta_{CF} - \upsilon_{T_S} T_{S,t} - CF_t\big)\, dt, \quad CF_0 > 0, \tag{3}$$

where $T_{S,t}$ denotes surface temperature (introduced below), θ_{CF} is the baseline level of climate-feedback effects absent surface warming, $\upsilon_{T_S} > 0$ measures how rising surface temperatures reduce the Earth's ability to absorb CO_2,[11] and $\kappa_{CF} > 0$ governs the speed of mean-reversion to the long-run feedback level $\theta_{CF} - \upsilon_{T_S} T_{S,t}$. In essence, higher surface temperatures accelerate the deterioration of carbon sinks, creating a positive feedback loop. Including time dependence in carbon sink capacity is crucial because standard linear EBMs often underestimate future atmospheric CO_2 concentrations. They fail to capture the ocean's rapid initial uptake followed by prolonged equilibration, a dynamic that can lead to higher long-run CO_2 levels Glotter et al. (2014); IPCC (2013).

Temperature dynamics. We model global temperature using a two-box system that distinguishes between surface temperature $T_{S,t}$ and deep-ocean temperature $T_{DO,t}$. This distinction is essential because the deep ocean acts as a vast heat reservoir with century-scale equilibration

fixed, implying a more static view of the oceans' and terrestrial systems' ability to absorb CO_2.

[11] For instance, warmer ocean water holds less dissolved CO_2.

times, creating substantial inertia in the climate system. The temperature dynamics are:

$$dT_{S,t} = \left(\alpha_{E_S} E_{S,t} - \kappa_{T_S} T_{S,t}\right) dt \ + \ \alpha_{T_{DO}} \left(T_{DO,t} - T_{S,t}\right) dt, \tag{4}$$

$$dT_{DO,t} = \kappa_{T_{DO}} \left(T_{S,t} - T_{DO,t}\right) dt, \tag{5}$$

where $\alpha_{E_S} > 0$ measures equilibrium climate sensitivity (similar to Matthews et al. (2009))—that is, by how much surface temperatures rise in response to a unit increase in atmospheric CO_2 concentration—$\kappa_{T_S} > 0$ is the surface temperature decay rate (capturing radiative heat loss to space), $\alpha_{T_{DO}} > 0$ controls the rate of heat transfer from the deep ocean to the surface, and $\kappa_{T_{DO}} > 0$ governs the rate at which the surface warms the deep ocean. Importantly, we allow $\kappa_{T_{DO}} \neq \alpha_{T_{DO}}$, which implies an asymmetric exchange of heat between the surface and deep ocean (often termed "ocean heat uptake saturation").[12] While the ocean initially moderates surface warming by absorbing a significant fraction of excess heat, its buffering capacity diminishes over longer timescales as the deep ocean gradually equilibrates. Many simple two-box climate models assume that $\alpha_{T_{DO}} = \kappa_{T_{DO}}$ because energy conservation assumes that the heat flux from the surface to the deep ocean must equal the heat flux from the deep ocean back to the surface. However, in more realistic climate models, the energy transfer between the surface and deep ocean is not symmetric due to factors like, different heat capacities of the ocean layers, vertical mixing processes and upwelling effects and the fact that heat transfer may be affected by radiative imbalances. The climate system modeled here directly influences economic outcomes by altering capital productivity, increasing the frequency of climate damages, and shaping incentives for mitigation and adaptation investments. These effects propagate through the economy, affecting production, investment, and policy choices. Having established the emissions and temperature dynamics, we now turn to the economic framework, which captures how households, firms, and governments respond to climate risks through investment and policy interventions.

3.2 Modeling the economy

Building on the climate dynamics established in the previous section, we now introduce the economic structure that governs production, investment, and policy responses to climate change. Our model features a multi-sector economy composed of a representative agent, a government, and three production technologies—clean, dirty, and residual—alongside investment in R&D. The representative agent chooses optimal consumption and allocates investments among these

[12] If $\alpha_{T_{DO}} > \kappa_{T_{DO}}$, the surface warms and cools faster than the deep ocean because the deep ocean has a higher heat capacity. This asymmetry better reflects real-world climate dynamics. (See Roe and Baker (2007), Gregory (2000), Winton et al. (2010), and Rugenstein et al. (2016)).

sectors. The government sets taxes and subsidies to adjust investment incentives and directs expenditures toward climate adaptation and carbon removal initiatives. Importantly, the government recognizes that clean capital investment is not yet sufficiently productive while also understanding that the damages from fossil fuel use exceed what is internalized by private agents.

3.2.1 Production economy: The status quo setup

We begin by characterizing the model economy under the status quo, or business-as-usual scenario, before introducing policy interventions in Section 3.2.3.

Aggregate wealth and capital structure. Let X_t denote aggregate wealth in the economy. In our framework, wealth equals the total capital stock:

$$X_t = K_t = K_{c,t} + K_{d,t}, \tag{6}$$

where $K_{c,t}$ is clean capital (e.g., renewable energy infrastructure), and $K_{d,t}$ is dirty capital (e.g., fossil-fuel-based production facilities). Aggregate output is linear in wealth:

$$Y_t = AX_t = AK_t, \tag{7}$$

where $A > 0$ is a constant productivity parameter calibrated to match observed global GDP (see Section 6.2).

Capital accumulation. The representative agent allocates investment flows $I_{c,t}$ and $I_{d,t}$ (measured in units of wealth per unit time) across the three capital types. Both capital stocks evolve according to:

$$\frac{dK_{j,t}}{K_{j,t}} = \mu_{K_j,t}\big(I_{j,t}/X_t\big)dt + \sigma_{K_j}\big(I_{j,t}/X_t\big)dW_{j,t}, \quad j \in \{c, d\}, \tag{8}$$

where $\mu_{K_j,t}(\cdot)$ is the expected net return (drift) on capital type j, $\sigma_{K_j}(\cdot) \geqslant 0$ governs the volatility of returns, and $W_{j,t}$ are independent standard Brownian motions capturing idiosyncratic productivity shocks to each sector. Both the drift and volatility depend on the investment share $\pi_{j,t} := I_{j,t}/X_t$. The net return on both capital types reflects gross productivity minus adjustment costs and depreciation. For clean capital, the net return is

$$\mu_{K_{c,t}} := \nu_{c,t}\,\frac{I_{c,t}}{X_t} - \left[\frac{\varphi_c}{2}\left(\frac{I_{c,t}}{X_t}\right)^2 + \delta_c\,\frac{I_{c,t}}{X_t}\right], \tag{9}$$

where $\nu_{c,t}$ is the time-varying productivity of clean capital (driven by R&D, as specified below), $\varphi_c > 0$ measures quadratic adjustment costs (reflecting frictions in reallocating resources to the clean sector), and $\delta_c > 0$ is the depreciation rate. For dirty capital, which benefits from fossil fuel usage, the net return is:

$$\mu_{K_{d,t}} := \nu_d \, FF \, \frac{I_{d,t}}{X_t} - \left[\frac{\varphi_d}{2} \left(\frac{I_{d,t}}{X_t} \right)^2 + \delta_d \, \frac{I_{d,t}}{X_t} \right], \tag{10}$$

where $\nu_d > 0$ is the productivity parameter for dirty capital, $FF > 0$ represents the fixed fossil fuel input intensity (measured in exajoules, calibrated to current usage levels), $\varphi_d > 0$ captures adjustment costs, and $\delta_d > 0$ is the depreciation rate.

R&D and endogenous clean productivity. A key feature of our model is that clean-sector productivity $\nu_{c,t}$ evolves endogenously through R&D investment. Let $I_{RD,t}$ denote the flow of R&D expenditures (in units of wealth per unit time) devoted to improving clean technology. The dynamics of clean productivity are:

$$d\nu_{c,t} = \left(\theta_{\nu_c} \; + \; \epsilon_{I_{RD}} \, \frac{I_{RD,t}}{X_t} \; - \; \delta_{\nu_c} \, \nu_{c,t} \right) dt \; + \; \sigma_{\nu_c} \, dW_t^{\nu_c}, \tag{11}$$

where $\theta_{\nu_c} > 0$ reflects autonomous (exogenous) growth in clean-sector productivity, $\epsilon_{I_{RD}} > 0$ measures the effectiveness of R&D spending in raising productivity (capturing learning-by-doing and knowledge spillovers), $\delta_{\nu_c} > 0$ is the depreciation rate of clean productivity (representing knowledge decay or obsolescence), and $W_t^{\nu_c}$ is a standard Brownian motion (independent of both $W_{c,t}$ and $W_{d,t}$) governing innovation risk, with volatility parameter $\sigma_{\nu_c} \geqslant 0$. This specification captures a fundamental tension: R&D spending competes with immediate capital accumulation but generates persistent productivity gains. The parameter θ_{ν_c} governs baseline productivity growth, while $\epsilon_{I_{RD}}$ determines how aggressively directed innovation can accelerate the clean transition. These parameters are calibrated to match observed trends in renewable energy costs (see Section 6.2). In Section 3.2.3, we introduce a temporary government R&D subsidy to correct for private underinvestment in clean innovation due to knowledge spillovers.

The productivity gap and the case for intervention. Grounded in current economic realities, we assume that the rate of return on dirty capital investment ($\mu_{K_{d,t}}$) currently exceeds that on clean capital investment ($\mu_{K_{c,t}}$). This productivity gap implies that, absent policy intervention, private agents will overinvest in dirty capital relative to the social optimum. The government's role, formalized in Section 3.2.3, is to deploy taxes, subsidies, and public investments to internalize the climate externality and accelerate the transition to clean energy.[13]

3.2.2 Preferences of the representative agent

The representative agent's preferences are modeled using recursive utility, which allows for a separation between risk aversion and intertemporal substitution—a flexibility not available in standard time-separable expected utility. This distinction is particularly important in climate-economy models, where agents must trade off near-term consumption against uncertain, long-run climate damages. Let V_t denote the continuation value of utility at time t, representing the agent's expected future welfare from time t onward. The recursive utility is defined as

$$V_t := \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s)\, ds \right], \tag{12}$$

where $\mathbb{E}_t[\cdot]$ denotes the expectation conditional on all information available at time t (including the state variables X_t, $\nu_{c,t}$, $E_{S,t}$, $T_{S,t}$, $T_{DO,t}$, etc.), C_s is consumption at time $s \geqslant t$, and $f(C, V)$ is the normalized aggregator function that combines instantaneous consumption utility with continuation value. The aggregator is given by:

$$f(C, V) = \frac{\beta(1-\gamma)V}{1-1/\psi} \left(\left(\frac{C}{((1-\gamma)V)^{\frac{1}{1-\gamma}}} \right)^{1-1/\psi} - 1 \right), \tag{13}$$

where $\beta > 0$ is the subjective discount rate (rate of time preference). A higher β means the agent is less patient and places *less* weight on future consumption relative to current consumption. By $\gamma > 0$, we denote the coefficient of relative risk aversion, measuring the agent's aversion to uncertainty over consumption levels, and $\psi > 0$ is the elasticity of intertemporal substitution (EIS), measuring the agent's willingness to substitute consumption across time in response to changes in investment returns. A higher ψ indicates greater flexibility in adjusting consumption plans when expected returns vary. The key advantage of recursive utility is that γ and ψ are independent parameters. In standard expected utility, they are constrained by $\psi = 1/\gamma$, which often leads to implausibly high risk aversion or implausibly low willingness to substitute intertemporally. In climate applications, this flexibility matters: high risk aversion (γ) encourages precautionary savings and aggressive mitigation to avoid tail risks, while high EIS (ψ) facilitates consumption smoothing despite fluctuating climate damages

[13] This implies that the three types of capital are perfect substitutes in production. While this may appear strong, Acemoglu et al. (2012a) argue empirically that clean and dirty inputs are highly substitutable, and thus a high elasticity of substitution is not unrealistic. Moreover, Hambel et al. (2024) also assume that clean and dirty capital are perfect substitutes. Whether clean and dirty capital are close substitutes relative to residual capital depends on the composition of the residual sector; our calibration to observed investment flows suggests the assumption is reasonable.

and investment opportunities. In our baseline calibration (Section 6.2), we set $\psi = 1$, which yields tractable closed-form solutions for optimal policies. This restriction implies logarithmic instantaneous utility but preserves the key feature of recursive utility: the separation of risk preferences from intertemporal substitution, allowing us to vary γ independently to examine sensitivity to climate risk aversion.

3.2.3 The government's optimization problem

We now characterize the government's optimization problem. In our framework, we use the term "centralized economy" interchangeably with the "social planner problem", following standard economic theory. This approach models the government as a benevolent planner that internalizes externalities and optimally allocates resources to maximize social welfare. The government sets taxes, subsidies, and public expenditures to correct distortions in private investment incentives, ensuring that the long-run economic trajectory accounts for environmental and intergenerational concerns.

Correcting the productivity wedge for clean capital. From a socially optimal perspective, the productivity of clean and dirty capital investments should be adjusted to reflect their true economic and environmental impacts. The government recognizes that current clean capital investment is not sufficiently profitable from a social welfare standpoint. Specifically, the profitability of clean technology investment from the government's perspective exceeds the private return in Equation (9). The government perceives the net return on clean capital as

$$\mu^G_{K_c,t} := \left(\nu_{c,t} + w_{\nu_c,t}\right) \frac{I_{c,t}}{X_t} - \left[\frac{\varphi_c}{2}\left(\frac{I_{c,t}}{X_t}\right)^2 + \delta_c \frac{I_{c,t}}{X_t}\right], \tag{14}$$

where the factor $w_{\nu_c,t} > 0$ represents a productivity wedge between clean and dirty capital investment. This wedge captures the social value of clean capital that private agents fail to internalize, including positive externalities from reduced future emissions, learning spillovers, and avoided climate damages. The optimal subsidy should be set to eliminate the productivity differential entirely. Formally, the optimal wedge $w^*_{\nu_c,t}$ solves

$$\mathbb{E}_t\left[\frac{\partial\left(dK_{d,t}/dt\right)}{\partial I_{d,t}} - \frac{\partial\left(dK_{c,t}(w^*_{\nu_c,t})/dt\right)}{\partial I_{c,t}}\right] = 0, \tag{15}$$

which ensures that the marginal returns to investment are equalized across sectors once social benefits are properly accounted for.

Temporary subsidy for clean R&D. Beyond correcting the static productivity wedge, the government also recognizes that there is insufficient clean innovation in the decentralized equilibrium. Knowledge spillovers and learning-by-doing externalities imply that the social return to R&D exceeds the private return. From the government's perspective, the evolution of clean productivity should follow

$$d\nu_{c,t} = \Big(\theta_{\nu_c} \; + \; \big(\epsilon_{I_{RD}} + G_{s,t}\big)\frac{I_{RD,t}}{X_t} \; - \; \delta_{\nu_c}\, \nu_{c,t}\Big)\, dt \; + \; \sigma_{\nu_c}\, dB_t^{\nu_c},$$
$$dG_{s,t} = -\kappa_{G_s} G_{s,t}\, dt, \quad G_{s,0} > 0, \tag{16}$$

where $G_{s,t}$ represents a temporary government subsidy to clean innovation, initially positive and decaying exponentially at rate $\kappa_{G_s} > 0$. The two subsidies introduced in this setup play complementary but distinct roles. The profit subsidy $w^*_{\nu_{c,t}}$ increases the immediate profitability of clean capital investment. This is primarily a short-run effect: absent continued innovation in the clean sector, the productivity advantage conferred by the subsidy erodes once the policy is removed or clean capital is eventually taxed. In contrast, the R&D subsidy $G_{s,t}$ jump-starts clean-sector innovation, generating persistent productivity growth through its effect on $\nu_{c,t}$. However, because the subsidy is temporary, its direct effect fades over time. In the long run, clean sector productivity converges to $\theta_{\nu_c} + \epsilon_{I_{RD}} I_{RD,t}/X_t$, determined by the steady-state level of privately funded R&D investment undertaken in the absence of government intervention.

Internalizing the emissions externality. The government is aware that the dirty sector's negative emissions externality is not internalized by private agents. While the profitability of dirty capital investment from the government's perspective remains as specified in Equation (10), the government recognizes that dirty capital generates emissions as shown in Equation (2), leading to atmospheric CO_2 accumulation and eventual climate damages. The representative agent, absent a corrective policy, does not account for these external costs when making investment decisions. The government corrects this distortion through carbon taxation, formally characterized in Section 4.

Government budget constraint and public expenditures. Government expenditures consist of three components: adaptation spending $G_{a,t}$, investment in carbon capture technologies $G_{cc,t}$, and other exogenous public expenditures $G_{o,t}$. The first two are endogenously determined as part of the optimal policy portfolio, while the third captures baseline government consumption unrelated to climate policy. The government operates under a balanced budget constraint:

$$G_t + \Pi_t = \sum_{j\in\{a,cc,s,o\}} G_{j,t} + \Pi_t = \mathcal{T}_t, \tag{17}$$

where $G_t = \sum_j G_{j,t}$ denotes total government expenditures, Π_t represents lump-sum transfer payments to the representative agent, and $\mathcal{T}_t$ is total tax revenue collected from carbon taxes, capital income taxes, and consumption taxes (fully specified in Section 3.2.4).[14]

Wealth dynamics with climate disasters. The evolution of aggregate wealth in the centralized economy incorporates climate-related damages through discrete jump shocks. Wealth evolves according to

$$\frac{dX_t}{X_{t-}} = \sum_{j\in\{c,d,r\}} \frac{dK_{j,t}}{K_{j,t}} - \frac{C_t + G_t + I_{RD,t}}{X_t}\,dt + \left(e^{-Z_X} - 1\right)dN_t^{\lambda_X} - f_a(G_{a,t})\Lambda_t\,dt, \tag{18}$$

where the first term captures capital accumulation across all three sectors, the second term reflects aggregate consumption and expenditures (private consumption plus government spending and R&D), and the final two terms represent climate damages. We now describe the damage mechanism in detail. Climate-induced economic disasters arrive according to a Poisson process $N_t^{\lambda_X}$ with time-varying intensity λ_t^X. This process is distinct from the emissions tipping point process $N_t^{\lambda_{E_S}}$ introduced in Section 3.1.2: the latter governs abrupt increases in atmospheric CO_2, while the former captures sudden macroeconomic shocks such as hurricanes, floods, droughts, and other extreme weather events. Each time a disaster occurs (that is, when $dN_t^{\lambda_X} = 1$), aggregate wealth falls by a random fraction $1 - e^{-Z_X}$, where $Z_X \sim F_Z(\cdot)$ is the jump size drawn from a gamma distribution with shape parameter α_{Z_X} and scale parameter β_{Z_X}. The term $\left(e^{-Z_X} - 1\right)$ in Equation (18) represents the proportional loss in wealth conditional on a disaster. Since $Z_X > 0$, this term is always negative, reflecting wealth destruction. The intensity of climate disasters depends on surface temperature. Specifically, we assume

$$\lambda_t^X = \bar{\lambda}_X \frac{T_{S,t}}{T_{S,0}}, \tag{19}$$

[14] In our baseline model, adaptation spending $G_{a,t}$ and investment in carbon capture technologies $G_{cc,t}$ are exclusively undertaken by the government. The private sector does not directly invest in these areas. This assumption reflects the idea that both adaptation and large-scale negative-emissions technologies exhibit characteristics of public goods, in which the benefits are widely distributed and not easily appropriable by individual agents. Unlike traditional models in which government spending is often treated as exogenous and unproductive, here it plays an active role in mitigating climate damage. Specifically, adaptation investments protect the economy from climate-related shocks, while carbon capture directly reduces atmospheric CO_2 concentrations. These policy instruments ensure that government expenditures yield tangible benefits for economic resilience and emissions reduction. In alternative formulations, one could allow private agents to invest in adaptation or carbon removal, for instance by introducing private-sector incentives for resilience infrastructure or carbon offset markets. However, in our baseline model, these remain public-sector responsibilities, ensuring that mitigation and adaptation policies are coordinated effectively within a centralized framework.

where $\bar{\lambda}_X > 0$ is the baseline disaster arrival rate (at the initial temperature $T_{S,0}$), and $T_{S,t}$ is the surface temperature at time t. Thus, higher surface temperatures increase the frequency of climate-related disasters proportionally. The expected instantaneous damage rate, expressed as a fraction of wealth per unit time, is

$$\Lambda_t \, dt := \mathbb{E}_t\Big[\big(e^{-Z_X} - 1\big) dN_t^{\lambda_X}\Big] = \mu_{Z_X} \bar{\lambda}_X \frac{T_{S,t}}{T_{S,0}} \, dt, \tag{20}$$

where $\mu_{Z_X} := \mathbb{E}\big[e^{-Z_X} - 1\big] < 0$ is the expected proportional wealth loss conditional on a disaster occurring. For a gamma-distributed jump size, this expectation can be computed in closed form as $\mu_{Z_X} = (1 + \beta_{Z_X})^{-\alpha_{Z_X}} - 1$. The quantity $\Lambda_t < 0$ thus represents the expected flow of climate damages, increasing linearly with surface temperature.

Adaptation investments and damage mitigation. The government can mitigate climate damages through adaptation investments, captured by the function $f_a(G_{a,t})$ in Equation (18). Adaptation spending enhances the resilience of infrastructure and production to extreme events, reducing the intensity of expected damages. For adaptation to be effective, the functional form must satisfy several economic properties. First, zero adaptation spending should yield zero benefits: $f_a(0) = 0$. Second, adaptation should exhibit positive but diminishing marginal returns: $f_a'(G_{a,t}) > 0$ and $f_a''(G_{a,t}) < 0$. Initial investments significantly reduce damages, but additional expenditures yield progressively smaller benefits. We adopt the functional form

$$f_a(G_{a,t}) = \nu_a \left(-\frac{1}{\Lambda_t} \frac{G_{a,t}}{X_t} \right)^{\xi_a}, \tag{21}$$

where $\nu_a > 0$ measures the baseline effectiveness of adaptation investment and $\xi_a \in (0, 1)$ is an elasticity parameter governing how adaptation benefits scale with expenditure. This specification has an important interpretation: if the government sets adaptation expenditures such that $f_a(G_{a,t}) = 1$, the economy is fully protected from climate-induced shocks in expectation. Formally, the expected net damages from climate events are neutralized:

$$ND_t := X_t \mathbb{E}_t\Big[\big(e^{-Z_X} - 1\big) dN_t^{\lambda_X}\Big] - X_t f_a(G_{a,t}) \Lambda_t = X_t \big(1 - f_a(G_{a,t})\big) \Lambda_t = 0. \tag{22}$$

Thus, adaptation policies provide a mechanism to dynamically manage risk exposure, reducing the adverse effects of extreme climate shocks while maintaining economic stability. The final damage term in Equation (18), $f_a(G_{a,t}) \Lambda_t \, dt$, represents, in absolute terms, the net expected cost of climate damages after accounting for adaptation, measured as a flow reduction in wealth per unit time.

Carbon capture investments and negative emissions. Carbon capture and negative emission technologies play a crucial role in climate policy by actively reducing the stock of atmospheric CO_2, rather than merely limiting new emissions. Unlike mitigation efforts that decrease ongoing emissions, carbon capture removes previously accumulated carbon, offsetting past emissions and complementing long-term stabilization goals. The effectiveness of carbon capture must satisfy key economic and physical principles. First, higher investment should lead to greater CO_2 removal, but with diminishing returns: $f'_{cc}(G_{cc,t}) > 0$ and $f''_{cc}(G_{cc,t}) < 0$. Second, investment should be responsive to atmospheric CO_2 levels. When concentrations are high, incentives for removal should be stronger. Conversely, as CO_2 approaches a predefined target level, investment should taper off to ensure efficient resource allocation. We adopt the functional form

$$f_{cc}(G_{cc,t}) = \nu_{cc}\left(\left(E_{S,t} - \bar{E}_T\right)\frac{G_{cc,t}}{X_t}\right)^{\xi_{cc}}, \tag{23}$$

where $\nu_{cc} > 0$ measures the effectiveness of carbon capture investments, $\xi_{cc} \in (0,1)$ is an elasticity parameter governing the responsiveness of CO_2 removal to investment, $E_{S,t}$ is the atmospheric concentration of CO_2, and $\bar{E}_T$ is the target level at which carbon removal efforts should phase out. This formulation ensures that carbon capture policies dynamically adjust to the evolving CO_2 stock while reflecting diminishing returns in large-scale deployment. It is important to note that this specification optimizes the investment flow based on effectiveness in reducing atmospheric CO_2, but does not explicitly account for constraints on cumulative geologic storage capacity, recently estimated at a prudent limit of 1,460 $GtCO_2$ (Gidden et al., 2025). We use unconstrained optimization to derive the optimal policy mix and subsequently analyze the physical feasibility of this pathway with respect to planetary limits in Section 7.

Two types of climate risk: emissions tipping points versus economic disasters. This modeling approach differs from the standard framework employed in climate-economy models such as DICE, where damages are built into a deterministic damage function that reduces aggregate output. In our setup, both the arrival of damages and the magnitude of each disaster are stochastic. Moreover, damages directly reduce the capital stock rather than merely diminishing current output, capturing the persistent effects of climate shocks on productive capacity. By using two distinct Poisson processes—emissions tipping points in Equation (2) and economic disaster shocks in Equation (18)—we capture the key difference between long-run climate shifts that alter emissions trajectories and short-run macroeconomic shocks that affect wealth and investment. This distinction allows us to analyze how optimal policy responses vary depending on whether the primary risk stems from gradual climate change, abrupt tipping points, or acute economic damages from extreme weather events.

The planner's optimization problem. Having specified the economic and climate dynamics, we now state the government's formal optimization problem. Let

$$V_t := V(X_t, \nu_{c,t}, G_{s,t}, E_{S,t}, T_{S,t}, T_{DO,t})$$

denote the continuation value at time t, representing the planner's expected future welfare conditional on the current state. The planner chooses consumption, investment across the three capital types, R&D expenditures, and government spending on adaptation and carbon capture to maximize the representative agent's recursive utility:

$$V = \sup_{\{C_u, I_{j,u}, I_{RD,u}, G_{a,u}, G_{cc,u}\}_{u \geqslant t}} \mathbb{E}_t\left[\int_t^\infty f\big(C_u, V_u\big)\, du\right], \tag{24}$$

where $f(C,V)$ is the aggregator function defined in Equation (13). The optimization is subject to the economic constraints:

$$\begin{aligned}
\frac{dX_t}{X_{t-}} &= \sum_{j\in\{c,d\}} \frac{dK_{j,t}}{K_{j,t}} - \frac{C_t + G_t + I_{RD,t}}{X_t}\, dt + \big(e^{-Z_X} - 1\big) dN_t^{\lambda_X} - f_a(G_{a,t})\Lambda_t\, dt,\\
d\nu_{c,t} &= \left(\theta_{\nu_c} + \big(\epsilon_{I_{RD}} + G_{s,t}\big)\frac{I_{RD,t}}{X_t} - \delta_{\nu_c}\,\nu_{c,t}\right) dt + \sigma_{\nu_c}\, dB_t^{\nu_c},\\
dG_{s,t} &= -\kappa_{G_s} G_{s,t}\, dt, \quad G_{s,0} > 0,
\end{aligned}$$

where $G_t = \sum_{j\in\{a,cc,s,o\}} G_{j,t}$ is total government expenditure, and the climate constraints:

$$\begin{aligned}
dE_{S,t} &= \Big\{\zeta_E\big(\eta_d\, u_t(\tau_{d,t})K_{d,t} + \bar{E}_N\big) - \big(\delta_{E_S} E_{S,t} + \xi_{CF}\, CF_t\big) - f_{cc}(G_{cc,t})\Big\}\, dt + \zeta_E Z_{FB}\, dN_t^{\lambda_{E_S}},\\
dT_{S,t} &= \big(\alpha_{E_S} E_{S,t} - \kappa_{T_S} T_{S,t}\big)\, dt + \alpha_{T_{DO}}\big(T_{DO,t} - T_{S,t}\big)\, dt,\\
dT_{DO,t} &= \kappa_{T_{DO}}\big(T_{S,t} - T_{DO,t}\big)\, dt.
\end{aligned}$$

The aggregate resource constraint is $Y_t = C_t + G_t + I_t$, where $I_t = \sum_{j\in\{c,d,r,RD\}} I_{j,t}$. The Hamilton-Jacobi-Bellman (HJB) equation associated with this problem is

$$0 = \sup_{C_t, I_{j,t}, G_{a,t}, G_{cc,t}} f(C_t, V_t) + \mathcal{A}V(X_t, \nu_{c,t}, G_{s,t}, E_{S,t}, CF_t, T_{S,t}, T_{DO,t}), \tag{25}$$

where $\mathcal{A}(\cdot)$ is the infinitesimal generator operating on the value function V, incorporating the drift, diffusion, and jump components from all state variables. We characterize the solution to this problem in Section 4.

3.2.4 The representative agent's problem

We now characterize the representative agent's optimization problem in the decentralized equilibrium. Unlike the government, which internalizes all externalities and optimizes social welfare, the representative agent takes government policies as given and maximizes private utility subject to after-tax returns and budget constraints. The agent does not account for the climate externality when making investment decisions. This section establishes the mapping between the centralized and decentralized problems, which forms the foundation for characterizing optimal taxes and subsidies in Section 4.

After-tax returns on capital. From the household's perspective, the profitability of investing in clean and dirty capital is affected by taxes and subsidies set by the government. The net after-tax return on clean capital becomes

$$\mu^H_{K_{c,t}} := (1-\tau_{K_c,t})\nu_{c,t}\,\frac{I_{c,t}}{X_t} - \Big[\frac{\varphi_c}{2}\Big(\frac{I_{c,t}}{X_t}\Big)^2 + \delta_c\,\frac{I_{c,t}}{X_t}\Big], \tag{26}$$

where $\tau_{K_c,t}$ is the capital income tax (or subsidy if negative) on clean investment. When $\tau_{K_c,t} < 0$, the government subsidizes clean capital to correct for the productivity wedge identified in Equation (14). Similarly, the after-tax return on dirty capital is

$$\mu^H_{K_{d,t}} := (1-\tau_{d,t})\nu_d\,FF\,\frac{I_{d,t}}{X_t} - \Big[\frac{\varphi_d}{2}\Big(\frac{I_{d,t}}{X_t}\Big)^2 + \delta_d\,\frac{I_{d,t}}{X_t}\Big], \tag{27}$$

where $\tau_{d,t} > 0$ represents the carbon tax imposed on dirty capital investment. This tax serves as the primary mechanism through which the government internalizes the emissions externality that the household ignores.

Household wealth dynamics. The evolution of household wealth differs from the government's formulation in Equation (18) in two key respects. First, the household pays taxes on consumption and capital income. Second, the household receives lump-sum transfers from the government to satisfy the balanced budget constraint. Household wealth evolves according to

$$\frac{dX_t}{X_{t-}} = \sum_{j\in\{c,d,r\}} \frac{dK_{j,t}}{K_{j,t}} - \frac{(1+\tau_{C,t})C_t + I_{RD,t} - \Pi_t}{X_t}\,dt + \big(e^{-Z_X}-1\big)dN_t^{\lambda_X} - f_a(G_{a,t})\Lambda_t\,dt, \tag{28}$$

where $\tau_{C,t}$ is the consumption tax at time t, Π_t denotes lump-sum transfer payments from the government, and the capital dynamics $dK_{j,t}/K_{j,t}$ reflect the after-tax returns specified in Equations (26) and (27). Note that the household benefits from government adaptation spending $G_{a,t}$ through the damage mitigation term $f_a(G_{a,t})\Lambda_t$, even though it does not directly control this policy instrument. The household treats the adaptation function and disaster intensity as exogenous environmental variables.

The household's optimization problem. The representative agent chooses consumption and investment across the three capital types to maximize recursive utility, taking as given all government policies, tax rates, and the evolution of climate variables. Formally, the household solves

$$V_t^H = \sup_{\{C_u, I_{c,u}, I_{d,u}, I_{r,u}, I_{RD,u}\}_{u \geqslant t}} \mathbb{E}_t \left[\int_t^\infty f\big(C_u, V_u^H\big)\, du \right], \tag{29}$$

subject to the wealth dynamics in Equation (28), the capital accumulation equations with after-tax returns, and the budget constraint $Y_t = C_t + I_t$. The clean productivity dynamics in Equation (11) and climate state variables $(E_{S,t}, T_{S,t}, T_{DO,t})$ evolve exogenously from the household's perspective, though they depend on aggregate outcomes. The associated Hamilton-Jacobi-Bellman equation is

$$0 = \sup_{C_t, I_{c,t}, I_{d,t}, I_{r,t}, I_{RD,t}} f(C_t, V_t^H) + \mathcal{A}^H V^H(X_t, \nu_{c,t}, G_{s,t}, E_{S,t}, CF_t, T_{S,t}, T_{DO,t}), \tag{30}$$

where $\mathcal{A}^H$ is the infinitesimal generator incorporating the after-tax dynamics of wealth and capital. Critically, the household does not optimize over adaptation spending $G_{a,t}$ or carbon capture $G_{cc,t}$, as these remain under government control.

Decentralization and competitive equilibrium. In the decentralized equilibrium, the representative agent's optimal choices must be consistent with the government's policy instruments. The government selects tax rates $\{\tau_{C,t}, \tau_{c,t}, \tau_{d,t}\}$ and public expenditures $\{G_{a,t}, G_{cc,t}, G_{o,t}\}$ such that the household's privately optimal decisions replicate the socially optimal allocations from the centralized problem. This requires the marginal conditions from the household's problem in Equation (30) to coincide with those from the planner's problem in Equation (25) once taxes and subsidies are appropriately chosen. We characterize the optimal policy instruments that achieve this decentralization in Section 4, where we derive explicit formulas for the carbon tax, clean capital subsidy, and government expenditures as functions of the state variables.

4 General equilibrium: An explicit solution

Having established the government's centralized optimization problem and the household's decentralized problem in Section 3.2, we now characterize the optimal climate policy mix. This section derives explicit solutions for consumption, investment, and government expenditures, and shows how to implement the social optimum through an appropriately designed system of taxes and subsidies. Our analytical approach exploits the recursive structure of the model under specific functional form restrictions, yielding closed-form policy rules that transparently reveal the economic forces governing optimal resource allocation across mitigation, adaptation, and carbon removal.

Solution method and key assumptions. We obtain explicit solutions under a set of simplifying assumptions that preserve the model's core economic mechanisms while ensuring tractability. First, we restrict the elasticity of intertemporal substitution to unity, $\psi = 1$. This restriction delivers logarithmic instantaneous utility and implies that the consumption-to-wealth ratio is constant, greatly simplifying the dynamic optimization. In this limiting case, the aggregator function becomes

$$\lim_{\psi \to 1} f(C, V) = \beta(1-\gamma)V \left(\log C - \frac{1}{1-\gamma} \log\big((1-\gamma)V\big) \right). \tag{31}$$

Second, we fix R&D investment expenditures as a constant share of wealth: $I^*_{RD,t} = \bar{I}_{RD} X_t$, where $\bar{I}_{RD}$ is calibrated to match observed global R&D spending on clean technologies. Third, we assume the jump size distributions for wealth shocks and emissions tipping points are gamma-distributed with shape and scale parameters α_ℓ and β_ℓ for $\ell \in \{X, FB\}$, allowing us to compute all jump-related expectations in closed form. Fourth, we set the curvature parameters for adaptation and carbon capture to one-half: $\xi_a = \xi_{cc} = 0.5$. This choice balances realism (diminishing returns are substantial but not extreme) with analytical convenience. Finally, for our baseline characterization, we set the temporary R&D subsidy to zero: $G_{s,t} = 0$, focusing on the steady-state policy mix after transitional subsidies have phased out.[15]

Value function and optimal policies. Under these assumptions, the social planner's value function takes the form

$$V_t = \frac{X_t^{1-\gamma}}{1-\gamma} e^{-\phi(Q_t)}, \tag{32}$$

$$Q_t := \phi_0 + \phi_{\nu_c,\ell}\nu_{c,t} + \phi_{K_d} K_{d,t} + \phi_{\nu_c,q}\nu_{c,t}^2/2 + \phi_{E_S} E_{S,t} + \phi_{CF} CF_t + \phi_{T_S} T_{S,t} + \phi_{T_{DO}} T_{DO,t},$$

[15] In the general case where $\psi \neq 1$, closed-form solutions are unavailable, but approximate analytical solutions can be derived using Campbell-Shiller linearization techniques (Campbell and Shiller, 1988), as employed in subsequent work by Chacko and Viceira (2005) and Drechsler (2013). These approximations involve first-order Taylor expansions around the unconditional mean of the state variables. Since only wealth exhibits jump risk while other state variables follow diffusive or deterministic processes, the accuracy of such approximations is high. We reserve this extension for future work, as our baseline calibration with $\psi = 1$ already captures the key policy trade-offs.

where the coefficients $\{\phi_0, \phi_{\nu_c}, \phi_{\nu_c^2}, \phi_{K_d}, \phi_{E_S}, \phi_{CF}, \phi_{T_S}, \phi_{T_{DO}}\}$ are determined by substituting this guess into the HJB equation (25) and matching coefficients.[16] Given this value function, optimal consumption and investment policies are obtained by maximizing the right-hand side of the HJB equation. The optimal consumption-to-wealth ratio from both the government's and household's perspectives is

$$\frac{C_t^{G,*}}{X_t} = \beta, \qquad \frac{C_t^{H,*}}{X_t} = \frac{\beta}{1+\tau_{C,t}}, \tag{33}$$

where the household's consumption is distorted by the consumption tax $\tau_{C,t}$. As is standard with logarithmic preferences, the consumption-to-wealth ratio is constant and determined solely by the time discount parameter β, provided that taxes are time-invariant.[17] Optimal investment shares across the three capital types reflect the standard trade-off between marginal productivity and adjustment costs, modified by taxes and subsidies. For clean capital, the optimal shares are

$$\frac{I_{c,t}^{H,*}}{X_t} = \frac{(1-\tau_{c,t})\nu_{c,t} - \delta_c}{\varphi_c}, \qquad \frac{I_{c,t}^{G,*}}{X_t} = \frac{\nu_{c,t} + w_{\nu_{c,t}} - \delta_c}{\varphi_c}, \tag{34}$$

The government's investment rule incorporates a productivity wedge $w_{\nu_{c,t}} > 0$, representing the social value of clean capital that private agents fail to internalize. For dirty capital, the optimal shares are

$$\frac{I_{d,t}^{H,*}}{X_t} = \frac{(1-\tau_{d,t})\nu_d FF - \delta_d}{\varphi_d}, \qquad \frac{I_{d,t}^{G,*}}{X_t} = \frac{\nu_d FF - \delta_d}{\varphi_d}, \tag{35}$$

The household's investment is distorted downward by the carbon tax $\tau_{d,t}$, which the government uses to internalize the emissions externality. The optimal government expenditures on adaptation and carbon capture are state-contingent, responding dynamically to climate conditions. Adaptation spending increases with disaster intensity:

$$\frac{G_{a,t}^*}{X_t} = -\Lambda_t \frac{\nu_a^2}{4}, \quad \text{where} \quad \Lambda_t := \mu_{Z_X} \bar{\lambda}_X \frac{T_{S,t}}{T_{S,0}}, \tag{36}$$

reflecting the fact that higher surface temperatures increase the frequency of climate disasters, necessitating greater investment in resilience. The parameter ν_a controls the effectiveness of adaptation; higher values imply both greater expenditure and more effective protection against climate shocks. Carbon capture investment increases with the gap between current and target

[16] The algebra is lengthy but straightforward; we relegate the derivation to Appendix A.

[17] If $\psi \neq 1$, the consumption-to-wealth ratio becomes state-dependent, varying with $\nu_{c,t}$, $E_{S,t}$, $T_{S,t}$, and other climate variables, reflecting precautionary savings motives in response to evolving climate risks.

atmospheric CO_2 concentrations:

$$\frac{G^*_{cc,t}}{X_t} = \left(\frac{\nu_{cc}\phi_{E_S}}{2(\gamma-1)}\right)^2 \left(E_{S,t} - \bar{E}_T\right), \tag{37}$$

where ν_{cc} measures carbon capture effectiveness and $\phi_{E_S} < 0$ is the value function coefficient on atmospheric CO_2, capturing the shadow cost of emissions. This policy converges to zero as $E_{S,t}$ approaches the target $\bar{E}_T$, ensuring that carbon removal efforts taper off once stabilization goals are achieved. Conversely, a lower target necessitates greater investment in negative-emission technologies, as achieving lower concentrations requires more intensive carbon capture.

These policy rules embody several key economic principles. Consumption smoothing dictates a constant consumption share under logarithmic preferences. Investment allocation balances productivity gains against adjustment costs and climate externalities. Adaptation spending scales with disaster risk, providing precautionary protection. Carbon capture targets the stock problem created by accumulated emissions. Together, these instruments form a comprehensive policy portfolio that addresses both the flow of new emissions and the stock of historical pollution.

4.1 Implementing the optimum: Taxes and subsidies

The policies characterized above solve the centralized planner's problem. We now show how to implement the social optimum in a decentralized economy through an appropriately designed tax-and-subsidy system. The government must set capital income taxes, consumption taxes, and sectoral subsidies such that households, acting in their private interest, make choices that coincide with the social optimum. This requires equalizing the optimal choice conditions from the household's problem (30) with those from the planner's problem (25).

Optimal tax rates. Comparing the optimal investment rules in Equations (34) and (35), we obtain the tax rates that decentralize the optimum. The optimal consumption tax is zero

$$\tau^*_{C,t} = 0, \tag{38}$$

implying that distorting the consumption–saving margin is suboptimal when capital income taxes are available. The optimal tax (or subsidy) on clean capital is

$$\tau^*_{c,t} = -\frac{w_{\nu_{c,t}}}{\nu_{c,t}}, \qquad w_{\nu_{c,t}} = \omega_s\left(\nu_d FF - \nu_{c,t}\right), \tag{39}$$

where $\omega_s > 0$ scales the magnitude of the wedge. When clean productivity is low relative to dirty productivity ($\nu_{c,t} < \nu_d FF$), we obtain $\tau^*_{c,t} < 0$, i.e. a subsidy that raises the after-tax return on clean investment. As R&D increases $\nu_{c,t}$ over time, the subsidy shrinks and may eventually turn into a tax once clean productivity exceeds dirty productivity. This captures the idea that the underlying market failure is temporary: once the technology gap is closed, there is no efficiency rationale for continuing to subsidize clean capital. Regarding the optimal carbon tax, as it is standard practice in the literature, we define the social cost of carbon (SCC) in wealth-equivalent terms as follows[18]

$$\mathrm{SCC}_t := -\frac{\partial V/\partial E_{S,t}}{\partial V/\partial X_t}, \tag{40}$$

and let $\tau_{d,t}$ denote the ad valorem tax on the return to dirty capital. Then, the unconstrained (first-best) ad valorem tax on dirty capital is

$$\tau^{FB}_{d,t} = \frac{1}{\nu_d}\frac{\mathrm{SCC}_t}{X_t} = \frac{1}{\nu_d}\frac{\eta_d \phi_{E_S}}{(1-\gamma)}. \tag{41}$$

As we will show when calibrating our model to key economic and climate moments, this first-best carbon tax in Equation (41) above is, in general, not admissible as it renders dirty capital investment non-profitable, i.e., in other words, the agent will want to optimally divest dirty capital. In the following Proposition, we formally define a first - and a second-best (admissible) carbon tax rate.

Proposition 1 (First- and second-best carbon tax and carbon price)**.** *We impose the following viability constraint on dirty capital investment:*

$$\big(1-\tau_{d,t}\big)\nu_d FF_t - \delta_d \;\geqslant\; 0, \tag{42}$$

Then, we define the implementable *(second-best) tax as a fraction* $\varsigma \in (0,1]$ *of the first-best rate, i.e.*

$$\tau^{SB}_{d,t} = \varsigma\, \tau^{FB}_{d,t} \tag{43}$$

subject to the viability constraint (42)*. Lastly, the model-implied carbon price in USD per ton*

[18] Because the carbon tax is expressed as a percentage, we normalize the social cost of carbon (SCC) by aggregate wealth X_t, i.e., we define the tax as $\tau_{d,t} = \mathrm{SCC}_t/X_t$. This makes $\tau_{d,t}$ dimensionless and removes the purely mechanical dependence of the tax on the scale of the economy: holding the environmental state and the capital composition fixed, a proportional change in X_t leaves $\tau_{d,t}$ unchanged. The normalization also renders the HJB problem homogeneous in wealth, which allows us to obtain a closed-form solution. Economically, it implies that the carbon tax does not mechanically rise with the absolute level of wealth, for example, when wealth increases because of a larger clean capital stock.

of CO_2 *is*[19]

$$P_{CO_2,t} = \tau_{d,t} \frac{r^{\text{pre}}_{d,t}}{\eta_d u_t} \times 10^3, \tag{44}$$

where $r^{\text{pre}}_{d,t} := \nu_d FF_t - \delta_d$ *denotes the pre-tax net rental rate on dirty capital.*

First, the viability constraint (42) rules out tax rates that would generate a strictly negative net return on dirty capital (and thus induce large-scale instantaneous divestment of the dirty capital stock). Second, as the proposition shows, ς quantifies the distance between the current policy mix and the idealized first-best benchmark. To see this, note that $\varsigma = 1$ corresponds to the unconstrained first-best allocation, in which the carbon tax coincides with the Pigouvian benchmark in (41). In contrast, $\varsigma < 1$ indicates that the first-best carbon tax is *not attainable* in our model because it would violate the viability condition (42) and push dirty capital investment into negative territory. Economically, ς summarizes a bundle of implementation frictions—irreversibility of dirty capital, concerns about abrupt scrapping and transition risk as well as political-economy constraints—that limit how aggressively the SCC can be translated into observed carbon prices. Moreover, ς characterizes a clear trade-off: A higher ς moves policy closer to the Pigouvian benchmark by raising the carbon tax, accelerating the phase-out of dirty capital and reducing long-run emissions. However, it also erodes the net return on dirty capital and tightens the viability constraint (42), increasing the risk of rapid capital scrapping and large transitional losses. A lower ς preserves dirty capital investment and smooths the transition, at the cost of slower decarbonization and higher cumulative emissions.

Revenue neutrality and the balanced budget. The government's balanced budget constraint in Equation (17) ensures that all expenditures are financed through tax revenues and lump-sum transfers. In the decentralized equilibrium, total tax revenue is

$$\mathcal{T}_t = \tau_{C,t} C_t + \tau_{c,t} \nu_{c,t} I_{c,t} + \tau_{d,t} \nu_d FF \, I_{d,t}, \tag{45}$$

which must equal government spending plus transfers:

$$\mathcal{T}_t = G_{a,t} + G_{cc,t} + G_{o,t} + \Pi_t. \tag{46}$$

Since $\tau^*_{C,t} = 0$ and the clean capital subsidy ($\tau^*_{c,t} < 0$) represents an expenditure rather than revenue, the carbon tax on dirty capital must be set sufficiently high to finance adaptation, carbon capture, and the clean subsidy, in addition to baseline government expenditures.

[19] The factor 10^3 arises because wealth is measured in trillions (10^{12}) of USD and emissions in gigatons of CO_2 (10^9 tons of CO_2).

Lump-sum transfers Π_t adjust residually to balance the budget. This structure implies that the carbon tax serves a dual purpose: correcting the emissions externality and generating revenue to finance the broader climate policy portfolio. In practice, revenue-neutral carbon tax reforms often recycle revenues through income tax cuts or per-capita dividends; our framework accommodates such arrangements via the lump-sum transfer Π_t, which can be reinterpreted as a rebate distributed to households.

The optimal policy system characterized here achieves the social optimum by aligning private incentives with social objectives. Households face after-tax returns that reflect the true social productivity of each capital type, accounting for climate externalities and innovation spillovers. Government expenditures on adaptation and carbon removal directly address public goods problems that markets cannot solve. Together, these instruments form a coherent policy framework that maximizes welfare subject to climate and resource constraints. The next section examines the quantitative implications of this policy mix by calibrating it to observed economic and climate data.

5 Policy Portfolio Analysis

Having characterized the optimal policy mix, we now examine how these policies jointly affect economic dynamics and climate trajectories. This section provides both analytical and quantitative insights into the mechanisms through which carbon taxation, clean subsidies, adaptation spending, and carbon removal interact to shape capital accumulation, emissions pathways, and welfare outcomes. We begin by analyzing the after-tax returns to capital investment, then examine adaptation's role in mitigating climate damages, and finally characterize how carbon capture policies affect atmospheric CO_2 dynamics.

After-tax capital returns and the clean transition. The optimal tax and subsidy system derived in Section 4.1 fundamentally alters the returns to clean and dirty capital investment. Substituting the optimal policies from equations 39 and 41 into the capital return equations yields the after-tax expected returns:

$$\mu^{AT}_{K_{c,t}} = \nu_{c,t}(1-\omega_s) + \omega_s \nu_d FF, \qquad \mu^{AT}_{K_d} = \nu_d FF - \frac{\eta_d \phi_{E_S}}{1-\gamma}. \tag{47}$$

The first expression reveals a critical policy trade-off. When $\omega_s = 1$, the planner fully reallocates productivity from dirty to clean capital through subsidies, rendering clean-sector innovation irrelevant for shaping investment returns. All clean capital receives the same effective return regardless of underlying productivity. Conversely, when $\omega_s < 1$, investment incentives become increasingly sensitive to the endogenous evolution of clean productivity $\nu_{c,t}$, creating a dynamic interaction between policy intervention and technological progress.

This interaction embodies a fundamental intertemporal trade-off. Initially, high subsidies (large ω_s) are necessary to overcome the clean sector's relative disadvantage and jump-start the transition. However, as R&D efforts raise $\nu_{c,t}$ over time, the need for subsidies diminishes, and the policy stance naturally shifts toward taxing clean capital once it becomes competitive. Allocating resources to R&D today reduces immediate capital accumulation but increases future productivity, thereby mitigating long-run policy costs. The speed and direction of the clean transition are therefore jointly governed by R&D effectiveness ($\epsilon_{I_{RD}}$) and subsidy intensity (ω_s). Excessive subsidization risks technological complacency, discouraging private innovation by making firms overly reliant on government support. Conversely, underinvestment in R&D delays the clean sector's ascent, prolonging the need for costly subsidies. Optimal policy ensures a smooth shift from subsidization to self-sustaining growth.[20]

For dirty capital, the after-tax return is reduced by the carbon tax, which internalizes the emissions externality through the term $\eta_d \phi_{E_S}/(1-\gamma)$. Higher emissions intensity (η_d) and greater climate damages (captured by $|\phi_{E_S}|$) both increase the carbon tax, reducing dirty capital investment. Risk aversion (γ) amplifies this effect because more risk-averse societies place greater weight on avoiding uncertain future climate damages.

Capital accumulation dynamics under alternative policies. Figure 2 illustrates the evolution of clean and dirty capital stocks under four policy scenarios: business-as-usual (no policy), subsidy only, R&D investment only, and the full optimal policy portfolio. Both capital types exhibit substantial inertia, particularly at the outset, indicating that policy changes must be substantial to measurably affect accumulation dynamics. This inertia is particularly pronounced for dirty capital, reflecting the long-lived nature of fossil fuel infrastructure and the slow pace of capital turnover.

Clean capital dynamics vary significantly across policy regimes. Adding a subsidy to clean productivity increases the growth rate of clean capital accumulation, but this increase is modest even over thirty years, i.e., the clean capital stock is not substantially higher than under business-as-usual, suggesting that static subsidies alone are insufficient to drive a rapid transition. R&D investment initially has only minor effects on clean capital accumulation

[20] Appendix A.4 analyzes how ω_s should vary depending on the long-run relative productivity differential between clean and dirty capital.

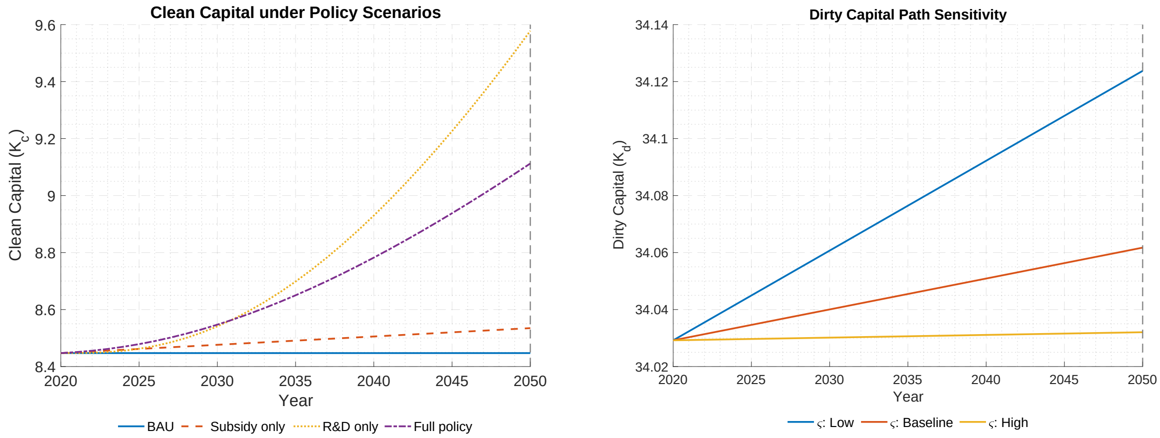


Figure 2: Clean and dirty capital accumulation over time. The left panel shows clean capital dynamics under four policy scenarios: business-as-usual (BAU), subsidy only, R&D investment only, and the full policy portfolio. The right panel shows dirty capital sensitivity to the carbon tax scaling parameter ς. All simulations use the baseline calibration from Section 6.2.

but generates exponential growth as time progresses. This pattern reflects the slow initial conversion (governed by the mean-reversion speed κ_{ν_c}) into more productive clean capital, followed by accelerating returns as productivity improvements compound.

Interestingly, the R&D-only policy eventually surpasses the full policy portfolio that combines subsidies with R&D investment. This somewhat counterintuitive result arises because the full policy includes only a temporary subsidy that becomes a tax once clean productivity $\nu_{c,t}$ exceeds dirty productivity $\nu_d FF$, i.e., $w_{\nu_c,t}$ becomes negative. While the subsidy is initially more effective at stimulating investment, it inadvertently disincentivizes clean innovation by making firms reliant on government support rather than productivity improvements. Once the subsidy transitions to a tax, the lack of prior innovation leaves clean capital less competitive, ultimately lowering long-run growth. This finding underscores the importance of designing temporary subsidies that phase out as innovation takes hold, rather than permanent transfers that create dependency.

Adaptation and expected net damages. Adaptation investments mitigate the economic impact of climate disasters by enhancing resilience to extreme events. Substituting the optimal adaptation policy $G^*_{a,t}$ from Equation (36) into the wealth dynamics and computing expected damages yields the net damage flow:

$$ND_t = \Lambda_t X_t \left(\frac{\nu_a^2}{2} - 1 \right), \tag{48}$$

where $\Lambda_t = \mu_{Z_X} \bar{\lambda}_X T_{S,t}/T_{S,0}$ is the expected damage intensity, increasing linearly with surface temperature. When there is no adaptation investment ($\nu_a = 0$), the formula reduces to the standard jump compensator: wealth decreases on average by $\Lambda_t X_t$ whenever a climate disaster strikes. With optimal adaptation, net damages are negative (adaptation more than offsets losses) whenever $\nu_a < \sqrt{2}$, and positive otherwise.[21] Our baseline calibration yields $\nu_a = 0.962 < \sqrt{2}$, implying that adaptation substantially reduces but does not fully eliminate expected damages. This reflects the empirical reality that even aggressive adaptation efforts cannot provide complete protection against extreme climate events.

Carbon capture and atmospheric CO_2 dynamics. Carbon capture technologies directly target the stock of accumulated atmospheric CO_2, complementing emission reductions from carbon taxation and clean subsidies. Substituting the optimal carbon tax τ_d^* from equation (A.12) and the optimal carbon capture investment $G_{cc,t}^*$ from Equation (37) into the climate dynamics yields the controlled evolution of atmospheric CO_2:

$$\begin{aligned} dE_{S,t} = \Big\{ & \zeta_E\big(\eta_d u(\tau_d^*) K_{d,t}(\tau_d^*) + \bar{E}_N\big) - \big(\delta_{E_S} E_{S,t} + \xi_{CF} CF_t\big) \\ & - \frac{\epsilon_{cc}^2}{2} \frac{|\phi_{E_S}|}{1-\gamma} \big(E_{S,t} - \bar{E}_T\big) \Big\} \, dt + \zeta_E Z_{FB} \, dN_t^{\lambda_{E_S}}, \end{aligned} \tag{49}$$

where $u(\tau_d^*) = \underline{u} + (1 - \underline{u}) e^{-\rho_u \tau_d^* / \bar{\tau}_d} < 1$ is the utilization rate under the optimal carbon tax. The carbon tax affects atmospheric CO_2 through two channels. The first-order effect operates through the utilization function: polluting firms reduce production capacity immediately in response to higher carbon prices, cutting back on operational intensity rather than waiting to retire capital. The second-order effect operates through the dirty capital accumulation equation:

$$\frac{dK_{d,t}^*}{K_{d,t}^*} = \frac{((1 - \tau_d^*) \nu_d FF - \delta_d)^2}{2 \varphi_d} \, dt, \tag{50}$$

which shows that the carbon tax reduces the growth rate of dirty capital, thereby gradually shrinking the emissions base. This two-pronged mechanism ensures that carbon taxation delivers both immediate and persistent emission reductions.

The carbon capture term in Equation (49) scales with the gap between current and target CO_2 concentrations, ensuring that removal efforts are most aggressive when concentrations are highest and taper off as stabilization goals approach. The effectiveness parameter ν_{cc} and the shadow cost $|\phi_{E_S}|$ jointly determine the intensity of carbon removal, with higher climate damages justifying greater deployment. Our calibration (Section 6.2) implies that carbon capture must scale from current levels of 0.04 $GtCO_2$/year to over 3.7 $GtCO_2$/year by 2050 to achieve the RCP4.5 temperature trajectory, underscoring the central role of negative emissions in achieving climate stabilization.

[21] When $\nu_a > \sqrt{2}$, adaptation investment not only protects wealth but may also help it grow over time. However, this must be weighed against the cost of adaptation, which is $G_{a,t}^*/X_t = \nu_a^2/4$. Thus, wealth increases in expectation if $\nu_a^2/4 > 1$, or equivalently $\nu_a > 2$, implying very high adaptation effectiveness.

Complementarities across policy instruments. Table 1 summarizes the primary targets, effectiveness, and limitations of each policy instrument. The analysis reveals strong complementarities across instruments that amplify their collective impact. Carbon taxation reduces new emission flows, decreasing the future burden on carbon capture technologies. Clean subsidies accelerate the transition away from fossil fuels, reducing both emissions and the economic costs of carbon taxation. Adaptation investments protect economic output from climate damages, sustaining tax revenues and enabling continued investment in mitigation and removal. Carbon capture addresses the stock problem created by historical emissions, which taxation and subsidies cannot remedy. These feedback effects imply that welfare gains from implementing a comprehensive policy portfolio substantially exceed the sum of individual policy effects—a manifestation of climate policy complementarity analogous to coordination gains in o-ring production functions.

The table underscores a central insight: no single instrument suffices to achieve climate stabilization at reasonable cost. Carbon pricing and subsidies address the flow of new emissions but cannot remove the historical stock. Adaptation manages unavoidable damages but does not prevent further warming. Carbon capture tackles the stock problem but cannot prevent new emissions. Only a comprehensive portfolio that coordinates all four instruments can achieve efficient climate stabilization while maintaining economic prosperity. The next section calibrates the model to observed economic and climate data, quantifying the welfare implications of alternative policy combinations and examining sensitivity to key structural parameters.

6 Model Calibration

We calibrate the model to replicate both observed economic aggregates and the climate projections implied by the Representative Concentration Pathway 4.5 (RCP4.5) scenario. This pathway represents a stabilization trajectory in which radiative forcing levels off at 4.5 W/m^2 by 2100, corresponding to approximately 2 °C of warming relative to pre-industrial levels. RCP4.5 provides a balanced benchmark between high-emission pathways such as RCP8.5 and stringent mitigation scenarios such as RCP2.6, representing a plausible middle ground consistent with current global policy commitments and technological trajectories. Our dual calibration strategy ensures that the model can simultaneously reproduce observed economic behavior and generate climate outcomes consistent with scientifically informed scenarios.

Table 1: Climate policy mix: primary targets, effectiveness, and limitations

Policy Instrument	Primary Target	Effectiveness and Limitations
(i) Carbon tax	New CO_2 emissions from dirty capital	Internalizes the emissions externality by raising the marginal cost of carbon-intensive production, thereby discouraging new emissions through both reduced utilization and lower investment. *Limitation:* Does not reduce the existing atmospheric CO_2 stock; may induce economic adjustment costs and short-run output losses if substitution toward clean capital is slow.
(ii) Subsidy to clean capital	Clean-to-dirty capital ratio in production	Encourages investment and technological adoption in the green sector, accelerating the structural transition away from fossil-based capital by correcting innovation externalities. *Limitation:* Does not directly reduce emissions from the dirty sector unless complemented by a carbon price; risks creating fiscal burdens and distorting innovation incentives if applied too aggressively or for too long.
(iii) Adaptation investment	Economic damages from climate change	Enhances resilience of output and capital stocks to higher temperatures and climate shocks, reducing the sensitivity of damages to climate variables and maintaining economic stability. *Limitation:* Does not affect the physical climate system—CO_2 concentration and temperature continue to rise—thus adaptation mitigates economic consequences rather than addressing root causes.
(iv) Carbon capture (CCUS)	Existing stock of atmospheric CO_2	Directly reduces the accumulated concentration of CO_2 in the atmosphere by removing and storing emissions, complementing flow-based mitigation policies and enabling ambitious temperature targets. *Limitation:* Technologically and energetically costly; large-scale deployment requires strong economic incentives, and its long-run feasibility is constrained by geologic storage capacity (estimated at approximately 1,460 $GtCO_2$).

6.1 Data sources and calibration targets

The calibration relies on climate data from the AR6 Climate Diagnostics, spanning the historical period from 1765 through long-term projections to 2500. Key climate inputs include global surface temperature anomalies from the CICERO Simple Climate Model (SCM), reflecting changes in global mean surface temperature relative to the pre-industrial baseline, and atmospheric CO_2 concentration levels measured in parts per million. These data establish the empirical link between anthropogenic emissions and global temperature dynamics, allowing us to discipline the climate component of our integrated assessment model.

The economic calibration targets both current estimates for 2020 and forward-looking projections for 2050, consistent with the RCP4.5 pathway. Global GDP in the baseline year amounts to approximately 85 trillion USD, and is projected to rise to about 200 trillion USD by 2050 according to World Bank estimates. The clean capital investment stands at 1.139 trillion USD in 2020 and is projected to increase to about 9.11 trillion USD by 2050, whereas dirty capital investment declines sharply from around 1.143 trillion USD to 0.67 trillion USD over the same period[22]. These two variables jointly capture the core reallocation mechanism of the energy transition and are therefore targeted in both years to ensure that the model reproduces an empirically plausible pace of decarbonization.[23]

Other variables are targeted primarily at their 2020 levels, serving as baseline reference points rather than explicit long-run policy goals. Clean energy subsidies amount to roughly 0.03 trillion USD per year (around 0.035% of global GDP), consistent with International Energy Agency estimates of direct fiscal support for renewables. The clean output share represents approximately 19.9% of total global production, reflecting a still-limited yet expanding role for the clean sector. Adaptation expenditures are around 63 billion USD annually, in line with estimates from the Global Commission on Adaptation[24]. Economic damages from climate-related events are benchmarked at roughly 0.27 trillion USD, based on global disaster loss data compiled by Munich Re and Swiss Re. Carbon capture deployment remains nascent, removing only 0.04 $GtCO_2$ per year in 2020.[25]

[22] See International Energy Agency (IEA) (2020)

[23] Specifically, GDP and sectoral investment levels are targeted for both 2020 and 2050, while variables such as subsidies, adaptation, damages, and carbon capture are matched only for 2020, as their long-run trajectories are determined endogenously by the model's policy dynamics.

[24] See Climate Policy Initiative (2021).

[25] See International Energy Agency (IEA) (2020).

Calibration of the utilization function. We model the impact of the carbon tax on the intensive margin of dirty capital through a smooth, bounded utilization function. For a given value-added carbon tax $\tau_{d,t}$, dirty-capital utilization is specified as

$$u_t(\tau_{d,t}) \;=\; u_{\min} \;+\; (1-u_{\min})\exp\Big(-\rho_u\,\frac{\tau_{d,t}}{r_d}\Big),$$

where $r_d = \nu_d FF - \delta_d$ is the pre-tax net rental rate on dirty capital, $u_{\min} \in (0,1)$ is a lower bound on utilization, and $\rho_u > 0$ governs the sensitivity of utilization to the tax. This specification guarantees that utilization is always between $u_{\min}$ and 1, equals 1 in the absence of a tax, and declines monotonically in the tax as a function of the *relative* tax burden $\tau_{d,t}/r_d$, rather than the absolute tax level. In the calibration, we set $u_{\min} = 0.1$, implying that even under very high carbon taxes, a residual 10% of dirty-capital services remain in use. This captures the idea that some fossil-based activities are difficult to eliminate entirely in the short to medium run and avoids a corner solution with an instantaneous shutdown of the dirty sector. The curvature parameter is calibrated to $\rho_u = 0.05$, which yields an elasticity of utilization with respect to the carbon tax of

$$\varepsilon_u \;=\; \frac{d\ln u}{d\ln \tau_{d,t}} \approx -0.60$$

at the baseline tax level. Economically, this means that a 10% increase in the carbon tax reduces dirty-capital utilization by about 6% on impact. This value is deliberately chosen to deliver a sizable but not extreme intensive-margin response: it is somewhat larger than typical short-run fossil-fuel demand elasticities (often around −0.2 to −0.3) but very close to average long-run price elasticities of gasoline and aggregate energy demand (around −0.5 to −0.6) documented in meta-analyses by Brons et al. (2008) and Labandeira et al. (2017). In this sense, the calibrated value $\varepsilon_u \approx -0.6$ can be interpreted as a conservative long-run benchmark for the responsiveness of dirty-capital utilization to carbon pricing.

6.2 Model Calibration Results

Table 2 reports the calibrated parameter values for both the climate and economic components of the model. We determine these values through a numerical optimization procedure that minimizes the squared distance between model-generated moments and their empirical counterparts, subject to parameter bounds that ensure economic and physical plausibility. The optimization jointly targets climate trajectories (CO_2 concentrations and temperature anomalies) and economic aggregates (GDP, investment flows, policy expenditures) over the 2020–2050 horizon.

Parameter	Value	Description/Range
Panel A: Climate Parameters (carbon cycle and two-box EBM)		
ζ_{ET}	0.137	CO_2 emissions scaling factor (tons → ppm)
δ_{ES}	0.004	Natural carbon removal rate [0.001, 0.01]
ξ_{CF}	0.261	Climate-feedback impact on carbon sinks (sink deterioration)
κ_{CF}	0.020	Climate-feedback mean reversion [0.01, 0.05]
θ_{CF}	0.173	Baseline climate-feedback level
υ_{T_S}	0.957	Sensitivity of feedbacks to surface temperature
α_{E_S}	0.005	Climate sensitivity (CO_2–temperature link)
α_{T_S}	1.316	Surface temperature adjustment / ocean–atmosphere coupling
$\alpha_{T_{DO}}$	0.508	Deep-ocean heat absorption / surface–deep exchange [0.3, 0.7]
$\kappa_{T_{DO}}$	0.091	Deep-ocean adjustment speed (multi-decade scale)
CF_0	6.347	Initial climate-feedback level
$T_{DO,0}$	0.087	Initial deep-ocean temperature (°C)
ES_{Target}	410.3	Target atmospheric CO_2 concentration (ppm)
FF	505.4	Fossil fuel input intensity (EJ)
Panel B: Economic Parameters (mitigation, adaptation, and risk)		
ν_{cc}	0.879	Carbon capture effectiveness
ν_a	0.962	Adaptation effectiveness
ω_s	0.747	Clean capital subsidy intensity
λ_{Z_X}	0.087	Baseline climate disaster arrival rate
α_{Z_X}	0.586	Disaster jump size (gamma shape)
β_{Z_X}	0.139	Disaster jump size (gamma scale)
$\epsilon_{I_{RD}}$	8.356	R&D effectiveness parameter
$\bar{I}_{RD}$	0.033	R&D investment share of wealth
κ_{ν_c}	0.075	Clean productivity mean reversion
θ_{ν_c}	0.017	Initial clean productivity level
ν_d	1×10^{-4}	Dirty capital productivity parameter
ρ_u	0.100	Utilization elasticity (carbon tax response)
ς	0.288	Carbon tax scaling (first-best to second-best)

Table 2: Calibrated climate and economic parameters: Panel A reports climate-system parameters governing atmospheric CO_2 dynamics and temperature evolution in the enhanced two-component energy balance model with time-varying carbon uptake (Section 3.1). Panel B reports economic parameters for mitigation and adaptation effectiveness, climate-disaster risk, and clean-technology dynamics. Ranges in square brackets indicate reasonable parameter bounds used in sensitivity analysis (Section 7). All parameters are calibrated to match RCP4.5 climate projections and contemporary economic aggregates.

Climate parameters. The calibrated climate parameters in Table 2 reflect plausible physical dynamics of the Earth system, with particular attention to carbon-cycle feedbacks and surface–ocean heat exchange. The CO_2 emissions scaling factor $\zeta_{ET} = 0.137$ maps fossil-fuel energy use into atmospheric carbon concentration increases, consistent with ranges in the DICE and PAGE models (0.10–0.15). The natural carbon removal rate $\delta_{E_S} = 0.004$ implies that roughly 0.4% of the atmospheric CO_2 stock is absorbed each year by natural sinks, primarily the oceans, terrestrial biosphere, and soil carbon pools. Although this figure may appear small, it is consistent with empirical estimates from carbon-cycle models, which suggest an effective atmospheric lifetime of CO_2 on the order of centuries.[26]

A central parameter is the climate sensitivity coefficient $\alpha_{E_S} = 0.005$, which translates atmospheric CO_2 concentration into radiative forcing. This value implies an equilibrium climate sensitivity well within the IPCC's "likely" range. In practical terms, even moderate increases in atmospheric CO_2 concentrations generate substantial and persistent surface temperature responses, reinforcing the urgency of mitigation. The feedback structure $(\xi_{CF}, \theta_{CF}, \kappa_{CF}) = (0.261, 0.173, 0.020)$ indicates a moderate positive feedback, meaning that rising temperatures gradually erode the capacity of natural sinks. Such feedback is empirically supported by recent CMIP6 evidence on reduced ocean carbon uptake and accelerated soil respiration.

The temperature dynamics parameters $\alpha_{T_S} = 1.316$, $\alpha_{T_{DO}} = 0.508$, and $\kappa_{T_{DO}} = 0.091$ describe the heat exchange between surface and deep ocean layers. These values imply that the deep ocean adjusts on decadal to multi-decade timescales, producing the well-known inertia of the global climate system. The temperature feedback coefficient $\upsilon_{T_S} = 0.957$ ensures that radiative forcing rises approximately linearly with surface temperature anomalies.

Economic parameters. Turning to the economic parameters, several values merit detailed discussion. First, the subsidy intensity parameter $\omega_s = 0.747$ implies that the magnitude of the profit subsidy is roughly three-quarters of the initial productivity differential between clean and dirty capital. This calibration corresponds to a comparatively strong, but still incomplete, policy intervention: the government substantially narrows the productivity gap through subsidies, while relying on directed R&D and capital reallocation to complete the transition endogenously. The parameters $\nu_{cc} = 0.879$ and $\nu_a = 0.962$ govern the effectiveness of carbon capture and adaptation expenditures, respectively, ensuring that plausible policy

[26] For comparison, values of $\delta_{E_S} = 0.003$–0.007 correspond to an atmospheric half-life of roughly 100–230 years, in line with estimates from the Bern carbon-cycle model and other Earth system simulations used in IPCC AR6; see Joos et al. (2013a); Canadell et al. (2021); IPCC (2021a).

efforts induce meaningful changes in atmospheric CO_2 and damages.

Second, the R&D parameters $(\bar{I}_{RD}, \epsilon_{I_{RD}}) = (0.033, 8.356)$ determine the intensity and productivity of directed innovation efforts. The investment share $\bar{I}_{RD} = 0.033$ implies that a bit more than 3% of aggregate output is devoted to R&D, consistent with upper-range estimates for global clean-energy R&D once private and public efforts are combined. Under these parameters, clean-sector productivity gradually rises from its baseline level, $\theta_{\nu_c} = 0.017$. At the estimated speed of mean reversion $\kappa_{\nu_c} = 0.075$, the half-life of deviations from the long-run productivity level is roughly 9 years $(\ln(2)/\kappa_{\nu_c})$, meaning that about half of any temporary gap between current and long-run clean productivity closes within a decade.[27]

Third, the disaster-risk parameters $(\lambda_{Z_X}, \alpha_{Z_X}, \beta_{Z_X}) = (0.0869, 0.5863, 0.1389)$ govern the frequency and severity of climate-induced economic shocks. At the current level in 2020 $(t = 0)$, the expected damage intensity is

$$\Lambda_0 = \lambda_{Z_X}\mu_{Z_X} = \lambda_{Z_X}\big(1 - (1 + \beta_{Z_X})^{-\alpha_{Z_X}}\big),$$

which corresponds to an underlying Poisson arrival rate $\lambda_{Z_X} \approx 0.087$, i.e., a climate disaster roughly every 10–12 years. The gamma-shaped jump-size distribution generates a right-skewed distribution of climate-related shocks, consistent with observed disaster frequencies and loss distributions. Conditional on a climate event occurring, around 7% of the agent's wealth is destroyed on average, which—using $Y_t = AX_t$ with $A \approx 2$—corresponds to about 3.5% of annual output per event. Multiplying by the arrival rate, the implied *unconditional* expected loss at the 2020 climate state is therefore on the order of $0.07 \times 0.0869 \approx 0.6\%$ of wealth, or roughly 0.3% of annual output.

When optimal adaptation is accounted for, expected damages are substantially reduced. Substituting the optimal adaptation policy $G^*_{a,t}$ into the wealth dynamics yields the net damage flow, which, using $Y_t = AX_t$, can be written as

$$\frac{ND_t}{Y_t} = \frac{\Lambda_t}{A}\left(\frac{\nu_a^2}{2} - 1\right).$$

In the absence of adaptation $(\nu_a = 0)$, expected damages at $t = 0$ are

$$\frac{ND_0}{Y_0} = -\frac{\Lambda_0}{A} \approx -0.32\% \text{ of annual output.}$$

[27]Such a speed of convergence is consistent with empirical estimates from technology diffusion studies in the energy sector, where learning-by-doing and knowledge spillovers yield relatively fast catch-up once innovation reaches scale; see, for example, Nordhaus (2014a); Grubb et al. (2021).

With the calibrated adaptation effectiveness $\nu_a = 0.962$, this falls to

$$\frac{ND_0}{Y_0} \approx -0.17\%,$$

so that optimal adaptation cuts expected climate-disaster losses by about 46% in 2020. As temperatures rise and Λ_t increases, both gross and net damages escalate, but the proportional mitigation effect of adaptation remains governed by the factor $1 - \nu_a^2/2 \approx 0.54$.

Finally, we highlight the calibration and interpretation of the carbon tax scaling parameter ς, which plays a central role in our analysis. In the frictionless planner's problem, the first-best Pigouvian tax implied by the shadow SCC is sufficiently high that, when translated into an ad valorem tax on dirty capital, it would drive the net return on dirty capital negative, thereby violating the viability constraint

$$(1 - \tau_{d,t})\,\nu_d F F_t - \delta_d \;\geqslant\; 0,$$

and inducing the instantaneous scrapping of the dirty capital stock. This constraint is binding, even if we make use of all four policies, so the implementable carbon tax must be a strict fraction of the notional first-best rate. We capture this wedge through $\varsigma \in (0, 1]$, as formalized in Proposition 1, such that

$$\tau_{K_d,t}^{SB} = \varsigma\,\tau_{K_d,t}^{FB}.$$

The estimated value $\varsigma = 0.288$ implies that, on average, the feasible carbon tax is somewhat below one third of the ideal Pigouvian level implied by the SCC. In the joint calibration, ς is disciplined by both climate targets (RCP4.5 CO_2 and temperature paths) and economic moments (the evolution of dirty investment and output), so that the resulting second-best carbon tax remains viable while the full portfolio of instruments (carbon tax, subsidies, adaptation, and carbon capture) delivers the desired transition.

6.3 Analyzing the model's fit

We now evaluate the model's ability to replicate both climate trajectories and economic aggregates. Figure 3 compares the model-implied paths for atmospheric CO_2 concentrations and surface temperature anomalies with the RCP4.5 benchmark projections from the AR6 Climate Diagnostics.

The model successfully reproduces both the short- and medium-run trajectories of atmospheric CO_2 and surface temperature, and converges to the long-run levels implied by the RCP4.5 scenario. This accuracy is crucial because the climate-economy problem is inherently

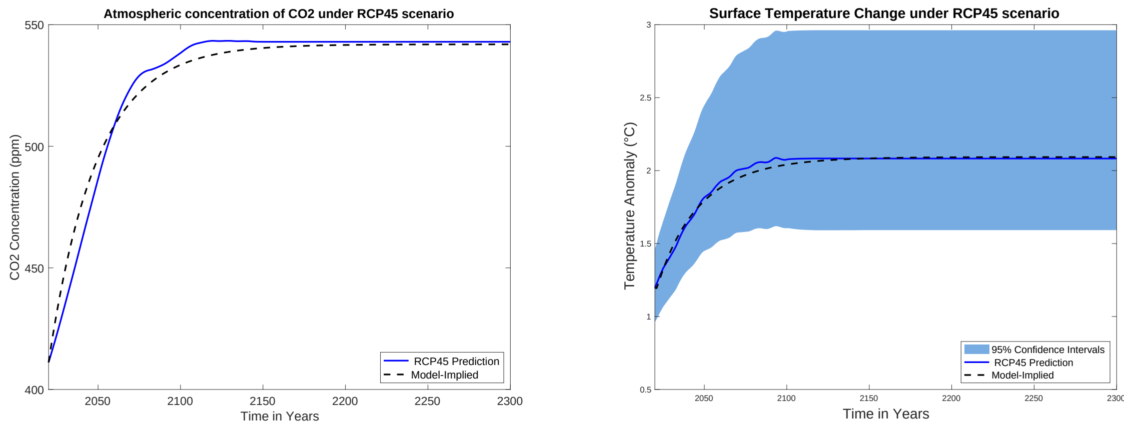


Figure 3: Model fit of atmospheric CO_2 concentrations and surface temperature anomalies under RCP4.5. The figure compares the model-implied trajectories (solid lines) with the benchmark scenario projections from the AR6 Climate Diagnostics (dashed lines). Atmospheric concentrations are reported in parts per million (ppm), whereas temperature anomalies denote deviations from the pre-industrial baseline in global mean surface temperature. Model results are calibrated to reproduce RCP4.5 trajectories for both carbon and temperature dynamics, using CICERO SCM benchmark data.

dynamic: the global climate system is far from any steady state and is in fact diverging toward a new and uncertain equilibrium. The model's ability to track RCP4.5 throughout the transition validates our climate block specification and parameter choices. Beyond climate dynamics, a central benchmark for evaluating the model is its ability to replicate key economic variables observed in the data. Table 3 summarizes model outcomes against empirical targets for 2020 and 2050.

Overall, the model reproduces both the direction and the scale of the transformations implied by RCP4.5 with high accuracy. It matches global GDP at both horizons, ensuring that the scale of the economy is consistent with observed and projected magnitudes, and generates carbon tax levels broadly in line with social cost of carbon estimates (around 200–300 USD per ton). The structural reallocation of investment from dirty to clean capital is also well captured: clean investment expands roughly fourfold, while dirty investment contracts substantially by mid-century, consistent with International Energy Agency (IEA) transition scenarios.[28] Adaptation expenditures rise over time, though they remain slightly below external benchmarks such as the World Bank and Global Commission on Adaptation projections, indicating that the model endogenously substitutes part of adaptation with mitigation and

[28]The negative value for the clean sector subsidy follows because in the long run, clean sector productivity outpaces dirty sector productivity, i.e. $\nu_d FF < \theta_{\nu_c} + \epsilon_{RD} I_{RD}$ which implies a negative subsidy $s_c = \omega_s(\nu_d FF - (\theta_{\nu_c} + \epsilon_{RD} I_{RD}))$. In other words, clean-sector profits will then be taxed in the same way as dirty profits.

	2020		2050	
	Data	**Model**	**Data**	**Model**
Global GDP (trillions USD)	85.27	85.73	200.00	199.45
Clean Subsidy (trillions USD)	0.0299	0.0295	—	—
Clean Output Share (%)	19.9	19.9	37.6	21.1
Clean Capital Investment (trillions USD)	1.3358	1.333	4.555	4.55
Dirty Capital Investment (trillions USD)	1.34	1.351	0.335	0.338
Adaptation Costs (trillions USD)	0.074	0.073	—	—
Economic Damages (trillions USD)	-0.314	-0.316	—	—
Carbon Capture ($GtCO_2$)	0.040	0.041	4.00	3.727
First-best carbon tax ($\varsigma = 1$, USD per ton CO_2)	177.92			
Second-best carbon tax ($\varsigma = 0.288$, USD per ton CO_2)	51.27			

Table 3: Model and data comparison for 2020 and 2050 projections. The table reports key macroeconomic aggregates, climate-related expenditures, and damages in trillions of USD (except where noted). The bottom rows show the implied first-best and implementable (second-best) carbon tax levels in USD per ton of CO_2, computed from the calibrated model.

carbon capture. Carbon capture deployment, in turn, reaches over 3.7 $GtCO_2$ annually by 2050—slightly below the central RCP4.5 projections.

7 Sensitivity Analysis of the Policy Portfolio

Having calibrated the model to match observed economic aggregates and RCP4.5 climate projections, we now examine how optimal policies respond to variation in key structural parameters. This sensitivity analysis serves two purposes. First, it reveals which parameters exert the strongest influence on policy design, helping to prioritize empirical research and identify critical sources of model uncertainty. Second, it illuminates the economic mechanisms through which different parameters affect optimal carbon taxation, adaptation spending, and carbon capture investment. We focus on four key parameters: the subjective discount rate β, adaptation effectiveness ν_a, the natural carbon sink rate δ_{E_S}, and the climate disaster arrival rate λ_{Z_X}. For each parameter, we consider a low case corresponding to a 25% reduction and a high case corresponding to a 25% increase relative to the baseline calibration in Table 2.[29]

[29] We focus on the subjective discount parameter β rather than risk aversion γ for two reasons. First, the climate literature discusses the impact of the subjective discount factor, rather than risk aversion, on optimal carbon taxes extensively, see Golosov et al. (2014b), Nordhaus (2014b), and Stern (2007). Second, as documented in Pindyck (2013a) and Hambel et al. (2021), increasing risk aversion paradoxically leads to less stringent climate policies. The mechanism operates through precautionary savings: higher risk aversion γ increases the value of wealth accumulation relative to mitigation and thereby reduces the optimal carbon taxes. In our framework, we observe a similar effect. However, an extension that is feasible, although would require

7.1 Carbon tax and surface temperature sensitivity

The carbon tax plays a central role in our policy framework, affecting both dirty capital accumulation through investment incentives and emissions intensity through the utilization function $u(\tau_{d,t})$. For this reason, we start by analyzing the sensitivity of the carbon tax and resulting surface temperature trajectories with respect to four key parameters. Figure 4 shows the percentage changes in the carbon tax given a $\pm 25\%$ in the above-mentioned parameter values. As the figure shows, the carbon tax is most sensitive to the arrival rate of climate-

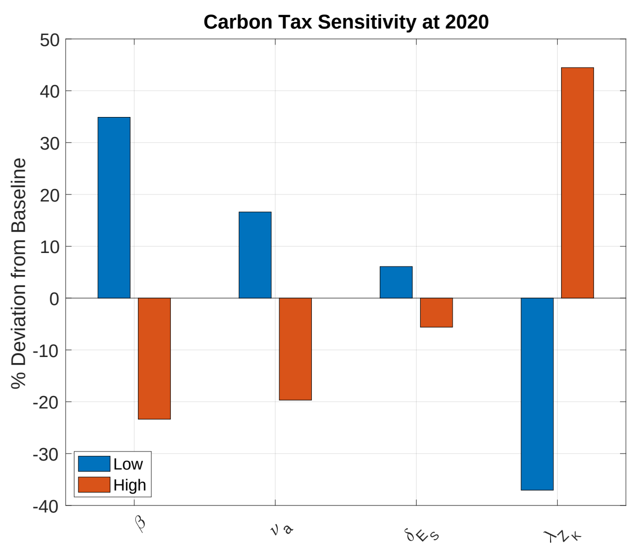


Figure 4: Carbon tax sensitivity to key parameters. The figure shows the percentage change in the carbon tax from its baseline level (\$178/tCO_2 from Table 3) when varying the parameters β, ν_a, δ_{E_S}, and λ_{Z_X} by $\pm 25\%$ from their baseline values in Table 2.

related disasters.[30] A 25% increase (decrease) in λ_{Z_K} increases (decreases) the carbon tax by 43% (38%). This result is very intuitive as when these climate-related disaster events become more frequent, the optimal carbon tax has to increase to reduce emissions further stringently. Next, changes in the subjective discount factor β exhibit a strong effect on the

approximation techniques, is to allow the elasticity of inter-temperporal substitution to be a free parameter rather than restricting it to unity as we do so in our setup. We leave this extension for future work as it significantly complicates the solution of our model.

[30]To better understand these comparative statics shown in Figure 4, recall that the optimal carbon tax is given by $\tau_d^* = \frac{1}{\nu_d} \frac{\eta_d \phi_{E_S}}{1-\gamma}$. Using Proposition A.1, we have $\frac{\partial \phi_{E_S}}{\partial \beta} < 0$, $\frac{\partial \phi_{E_S}}{\partial \nu_a} < 0$, $\frac{\partial \phi_{E_S}}{\partial \delta_{E_S}} < 0$ and $\frac{\partial \phi_{E_S}}{\partial \lambda_{Z_X}} > 0$. Thus, for example, given $\gamma > 1$, the carbon tax increases as the agent becomes more patient (i.e., lower β) because future climate damages carry greater weight. Analogously, if the arrival rate of climate-related disasters λ_{Z_X} increases, Proposition A.1 implies $\frac{\partial \phi_{E_S}}{\partial \lambda_{Z_X}} < 0$, so the carbon tax rises. The mechanism is the same as above: stronger disaster risk tightens the optimal policy, which depresses dirty input use and investment and thus reduces temperatures relative to the baseline.

carbon tax, i.e., when the representative agent becomes more impatient (higher β), then the optimal carbon tax declines as the agent places less weight on future consumption. Similarly, although with lower magnitudes, a higher adaptation effectiveness lowers the carbon tax as well - if every dollar invested in adaptation increases the resilience of the economy against climate disasters, there is less need for a stringent carbon tax and more room for usage of polluting inputs. A 25% change in nature's carbon sink rate δ_{E_S} alters the carbon tax only slightly, by about 5%. Next, we quantify how a 25% change in these parameters, except for the adaptation efficiency parameter ν_a, which we substitute by the utilization parameter ρ_u, changes the surface temperature predictions. Figure 5 summarizes the results

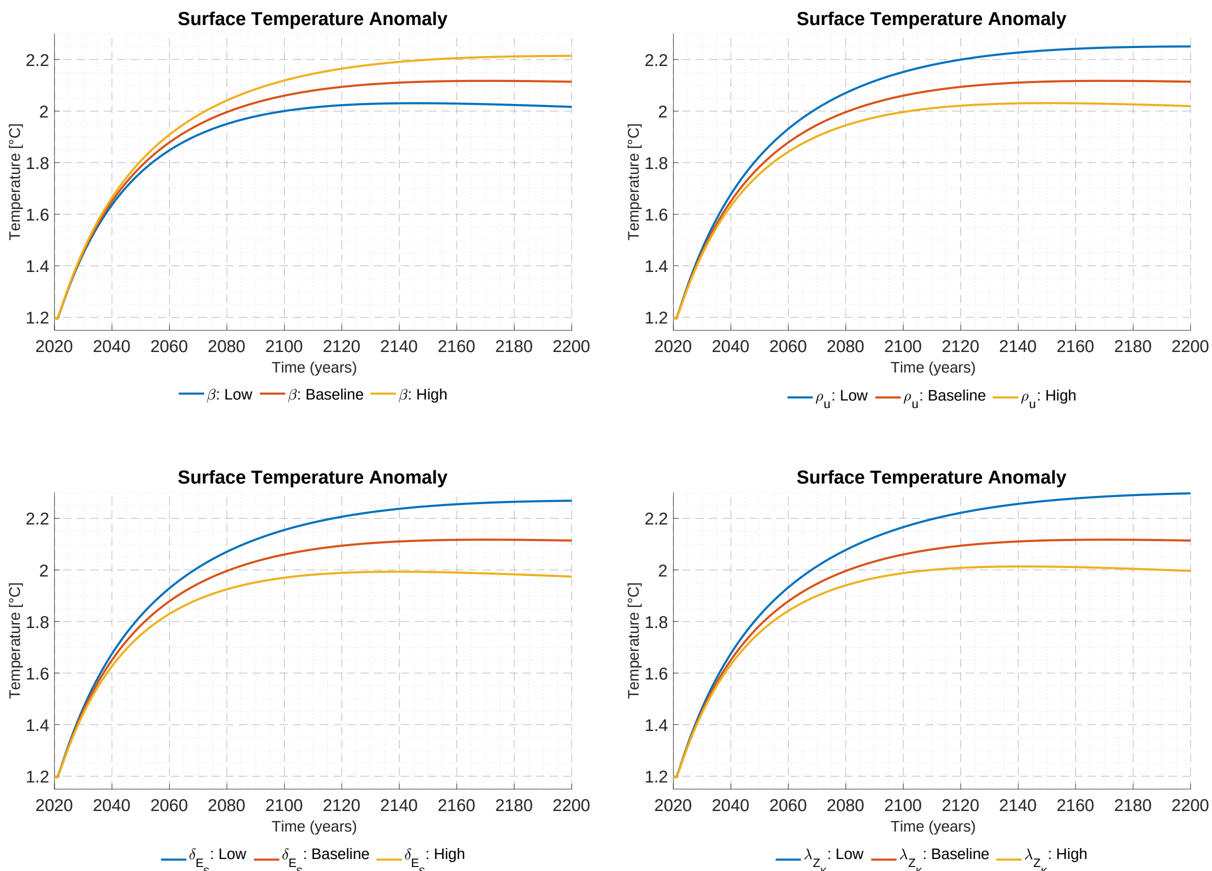


Figure 5: Carbon tax and surface temperature sensitivity. The four panels show surface temperature trajectories under ±25% variations in β (top left), ρ_u (top right), δ_{E_S} (bottom left), and λ_{Z_X} (bottom right). Solid lines represent the high case (+25%) and dashed lines represent the low case (−25%) relative to the baseline calibration in Table 2.

First, Figure 5 shows that, across all parameter perturbations, the induced changes in the global surface temperature anomaly are of similar magnitude and remain relatively modest.

This indicates substantial inertia in our climate framework: even sizeable shifts in economic primitives translate only gradually into temperature responses over the 2020–2050 horizon. Second, turning to the individual parameter sensitivities, Figure 5 shows that when the representative agent becomes more impatient - reflected by an increase in the subjective discount rate β - the carbon tax declines. As a consequence, surface temperatures rise modestly (again, see the top-left panel), since a lower carbon tax leads to greater utilization of carbon-intensive dirty capital. Changes in the utilization parameter ρ_u generate a similar qualitative pattern. A 25% increase in ρ_u leads to a roughly 5% decline in the global surface temperature anomaly. This effect is intuitive: for a given carbon tax, higher ρ_u makes utilization more elastic so that the same tax induces a stronger reduction in the use of polluting inputs. Next, increases in the natural carbon sink rate δ_{E_S} and in the climate disaster arrival rate λ_{Z_X} generate somewhat larger temperature responses than the discount-rate and utilization shocks. While the effect of a higher λ_{Z_X} affects surface temperatures through its impact on the carbon tax, changes in the natural carbon sink rate δ_{E_S} are more subtle, because δ_{E_S} affects both the carbon tax and the CO_2 dynamics directly. A higher δ_{E_S} increases the capacity of natural sinks to absorb emissions, which directly lowers atmospheric CO_2 for given emissions flows (see Equation (2)). At the same time, the improved sink capacity reduces the marginal value of abatement, which lowers the optimal carbon tax and tends to increase emissions. The overall effect on global surface temperatures is therefore a priori ambiguous. In our calibrated model, however, the direct sink effect dominates: the increase in δ_{E_S} reduces atmospheric CO_2 sufficiently strongly that temperatures fall, despite the weaker carbon tax effect. This is precisely what is visible in Figure 5.

7.2 The role of carbon capture technologies

A key finding from our analysis is that achieving climate targets under RCP4.5 requires not only carbon taxation but also large-scale deployment of carbon dioxide removal (CDR) technologies, particularly carbon capture and storage.[31] Moreover, given that fossil fuel use will decline only gradually in the coming decades, accelerated investment in carbon capture becomes essential to offset residual emissions and reduce atmospheric concentrations (Minx et al., 2018; Realmonte et al., 2019). Figure 6 illustrates the effect of varying two central policy levers: the effectiveness of carbon capture (ν_{cc}) and the target atmospheric concentration ($\bar{E}_T$).

[31] The central challenge is the persistently high stock of atmospheric CO_2 ($E_{S,t}$), which reflects the legacy of past emissions and the gas's long atmospheric residence time of decades to centuries (Archer, 2009; Joos et al., 2013a). Because CO_2 decays only slowly once emitted, policies relying solely on carbon pricing will be insufficient to meet temperature stabilization goals (Fuss et al., 2018; IPCC, 2022).

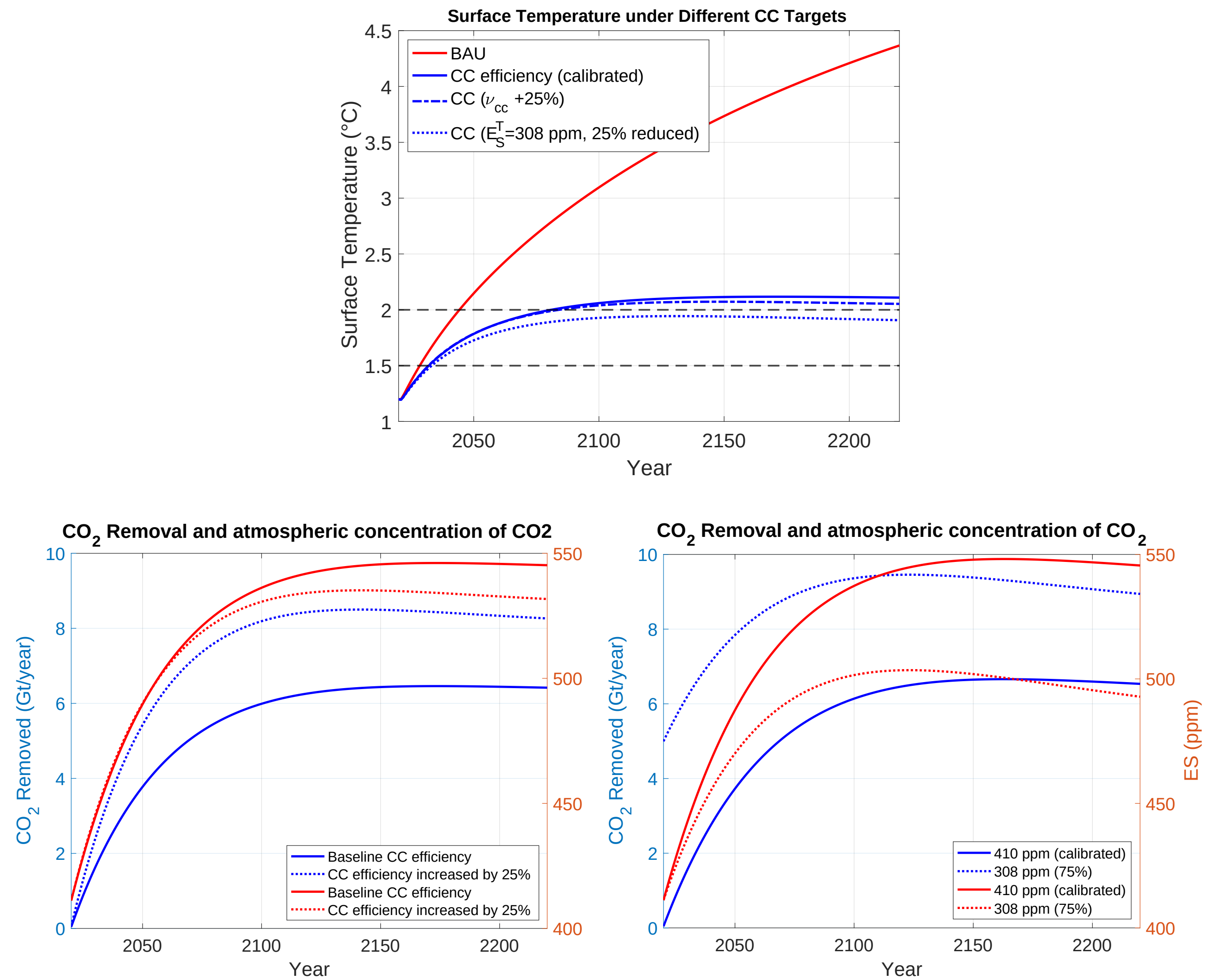


Figure 6: Carbon capture technologies and temperature trajectories. The top panel shows model-implied temperature projections under varying carbon capture effectiveness (ν_{cc}) and target atmospheric concentration ($\bar{E}_T$). The bottom two panels show the associated carbon removal measured in $GtCO_2$ per year (left) and the implied pathway of atmospheric CO_2 concentration (right). Model parameters are from Table 2. The business-as-usual (BAU) case corresponds to a scenario without carbon taxes, clean energy subsidies, adaptation, or carbon capture. Dashed gray lines in the top panel indicate temperature increases of 1.5 °C and 2 °C above pre-industrial levels.

In the top panel, the business-as-usual case without any policy intervention generates a surface temperature path resembling the RCP8.5 benchmark, with projected global temperatures rising rapidly and approaching 3 °C by 2100 relative to 1986–2005 levels.[32] Our benchmark

[32]RCP8.5 projections suggest a likely range of 2.8–4.5 °C by 2100.

policy calibration (blue solid line) limits warming to just above 2 °C. A 25% increase in carbon capture effectiveness ν_{cc} reduces long-run temperatures by approximately 0.1 °C relative to the RCP4.5 pathway. Moreover, decreasing the target level for the atmospheric concentration by 25% (from 410 ppm to 308 ppm) shows that the surface temperature increase stays below the 2 °C Paris target. The bottom panels reveal the associated scale of carbon removal. Under the baseline calibration (solid blue line, bottom-left panel), carbon capture rises from current levels of approximately 0.04 $GtCO_2$/year to about 3.7 $GtCO_2$/year by 2050, eventually reaching almost 6.5 $GtCO_2$/year by 2100 before starting to decline as atmospheric concentrations stabilize (left y-axis). If ν_{cc} increases by 25% (blue dashed line), carbon capture deployment accelerates dramatically, reaching approximately 8.2 $GtCO_2$/year by 2100 (dashed blue line) before falling slightly through the next century. While the effect of an increase in ν_{cc} is immediate on the captured amount of carbon dioxide, the associated concentration of CO_2 in the atmosphere remains comparable to the calibrated scenario until roughly the year 2085, before it approaches 530 ppm compared to 545 ppm under the RCP4.5 scenario (red lines left y-axis).

In the bottom-right panel, we show how the carbon removal and the atmospheric concentration of CO_2 change in response to a change in the E_S target level. Decreasing $\bar{E}_T$ by 25% triggers an immediate increase in carbon removal to about 5 $GtCO_2$/year, with deployment peaking around 9 $GtCO_2$/year by 2100 (dashed blue line, left y-axis). This aggressive removal pathway has noticeable effects on atmospheric CO_2: concentrations peak around 510 ppm in 2100 before declining below the 500 ppm level by 2250 (dashed red line), considerably lower than the 545 ppm concentration under the baseline RCP4.5 scenario (solid red line).

7.3 Expected economic damages

We now examine the sensitivity of expected economic damages to our four key parameters. Figure 7 summarizes the effects on both current (2020) and future (2080) expected damages, as well as cumulative damages over the transition period.

The top panel reveals that expected economic damages are most sensitive to the adaptation effectiveness parameter ν_a and the climate disaster arrival rate λ_{Z_X}, while the agent's subjective discount factor β and the carbon sink parameter δ_{E_S} have negligible effects on both current and future damages. Adaptation effectiveness has the largest impact: a 25% increase (decrease) in ν_a reduces (increases) expected damages by 48% (37%). The bottom panel shows the projected cumulative damages measured as a percentage of global GDP when

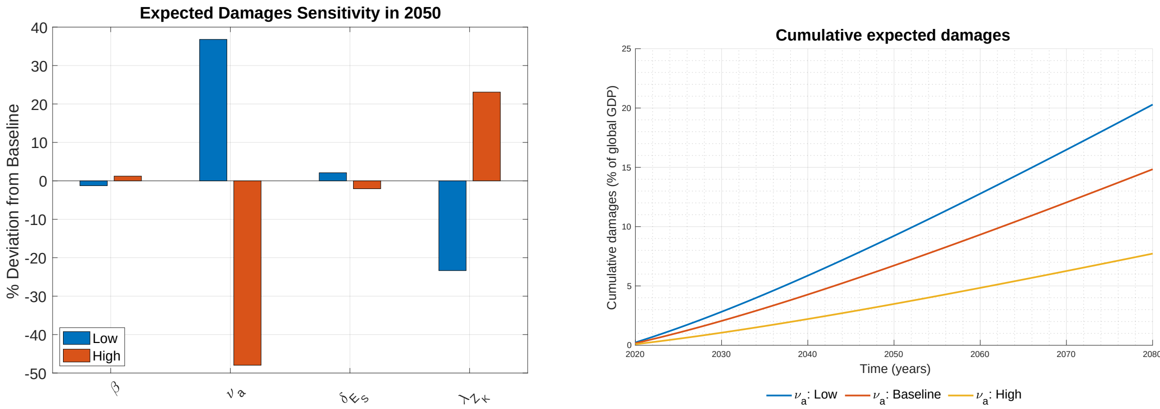


Figure 7: Economic damages: Sensitivity analysis. The top panel shows the percentage change in expected economic damages (measured in trillions of USD) for 2050 when varying key parameters by ±25%. The bottom panel shows projected cumulative damages over 2020–2080 measured as a percentage of cumulative global GDP under different levels of adaptation effectiveness ν_a.

varying adaptation effectiveness. The differences are staggering: under low adaptation effectiveness (ν_a reduced by 25%), cumulative expected economic damages are slightly more than 20% of global GDP over the 2020–2080 period, whereas they remain at around 7.5% if ν_a increases by 25%. To understand this pronounced sensitivity, recall that net expected damages expressed as a percentage of GDP are given by

$$\frac{D_t}{Y_t} = \frac{\Lambda_t}{A}\left(1 - \frac{\nu_a^2}{2}\right) = \frac{\mu_{Z_X}\bar{\lambda}_X T_{S,t}}{AT_{S,0}}\left(1 - \frac{\nu_a^2}{2}\right), \quad \mu_{Z_X} := \mathbb{E}_t\big[e^{-Z_X} - 1\big], \tag{51}$$

where economic damages increase linearly in the disaster arrival rate λ_{Z_X} but decrease quadratically in adaptation effectiveness ν_a. Hence, changes in ν_a have an overproportional effect on expected damages. However, this is not the only channel through which ν_a impacts damages. The adaptation effectiveness parameter also significantly affects the carbon tax (as shown in Figure 4), which in turn reduces new CO_2 emissions through both lower dirty capital investment and reduced utilization of existing dirty capital. This indirect effect slows the increase in global surface temperatures $T_{S,t}$, further reducing damages. Thus, adaptation effectiveness influences economic outcomes through two channels: a direct channel that reduces damage intensity conditional on temperature, and an indirect channel that operates through the carbon tax to reduce temperature itself. This dual mechanism explains why adaptation emerges as such a critical policy lever in our framework.

7.4 Achieving the 2 °C limit: Carbon tax versus carbon capture investment

In this subsection, we examine how carbon taxes and carbon capture investment can be deployed individually to limit global surface temperature increases to 2 °C above pre-industrial levels. We focus on these two instruments because they directly affect the stock of atmospheric CO_2: carbon taxes reduce emissions flows, while carbon capture removes accumulated stock.[33]

To isolate the direct effect of each policy instrument, we consider two counterfactual experiments. In the first, we vary the carbon tax while shutting down carbon capture; in the second, we vary carbon capture investment while shutting down the carbon tax. In both experiments, the clean-capital subsidy and adaptation expenditures are held fixed at their calibrated levels. We then ask: How high must the active policy (either the carbon tax or carbon capture) be to achieve the 2 °C target? [34] While analyzing each policy, we set the other to zero.[35] Figure 8 compares these two single-instrument scenarios.

The top panel of Figure 8 reveals that a policy without a carbon tax (dotted blue line) is less effective in reducing the surface temperatures compared to the policy without investment into carbon capture technologies (dashed black line). However, as the bottom-left panel shows, the reason for this result is that the required first-best carbon tax needs to increase by more than a factor of 2.5 relative to its baseline value of \$178/tCO_2, reaching approximately \$474/tCO_2. Similarly, the implementable second-best carbon tax increases 4 fold, from its baseline value 51 to 214 USD per ton of CO_2. This relatively large increase reflects two mechanisms. First, in our calibrated model, the dirty capital stock continues growing at the baseline carbon tax, implying rising future CO_2 emissions. Second, absent carbon capture investment (which we set to zero for this comparison), emissions reductions must come entirely from curtailing dirty capital investment and, more importantly, utilization, requiring a prohibitively high tax.

The bottom panel examines the alternative single-instrument approach: achieving 2 °C through carbon capture alone, with carbon taxes set to zero. Annual carbon removal would

[33] The other two policies—adaptation and clean capital subsidies—do not directly affect atmospheric CO_2 concentrations. Adaptation reduces economic damages conditional on temperature but does not alter the climate trajectory. Clean capital subsidies accelerate the transition away from fossil fuels but operate slowly through capital reallocation rather than immediately affecting emissions.

[34] Throughout this policy analysis, the subsidy and adaptation policies are not re-optimized but kept at their benchmark calibrated values.

[35] In practice, this means comparing two counterfactual scenarios: (i) carbon tax only, with $G_{cc,t} = 0$ and $\tau_{d,t}$ chosen to achieve 2 °C, and (ii) carbon capture only, with $\tau_{d,t} = 0$ and $G_{cc,t}$ chosen to achieve 2 °C. For the carbon capture analysis, we simultaneously change both ν_{cc} and $\bar{E}_T$ since by changing only one parameter, the model implies that it is not feasible to reach the 2 °C target.

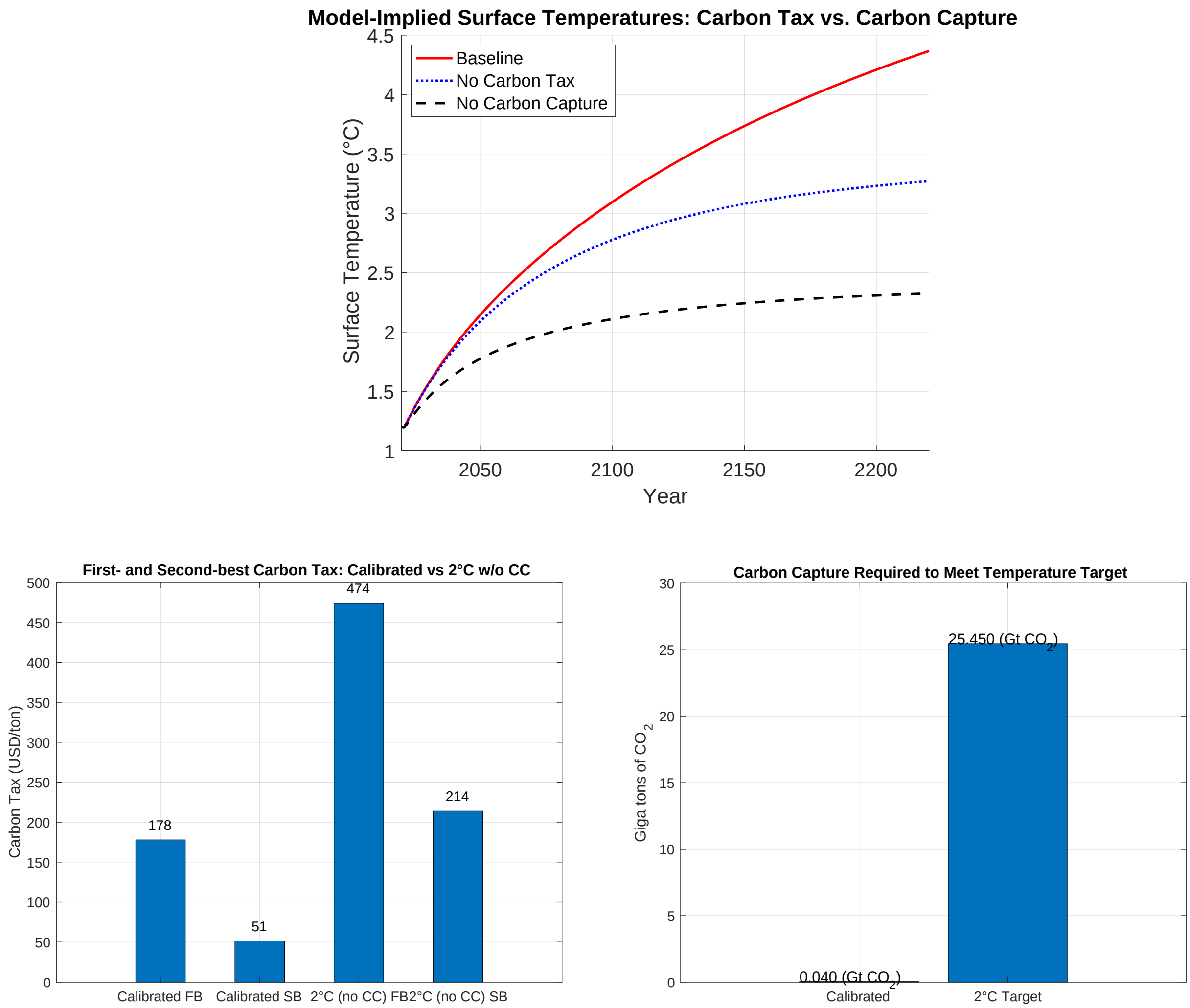


Figure 8: Policy analysis for achieving the 2 °C target. The top panel shows three scenarios: First, the business-as-usual scenario in which no policy instruments are being used, i.e., no carbon tax ($\varsigma = 0$), no subsidies ($\omega_s = 0$), no adaptation investment ($\nu_a = 0$), and no carbon capture investment ($\nu_{cc} = 0$). The dashed blue line (labeled "without carbon tax") shows the evolution of surface temperatures when all the policies but the carbon tax is used. Similarly, the dashed black line (labeled "without CC") shows the evolution of surface temperatures when all the policies but the carbon capture (CC) technologies are used. The bottom left panel compares the calibrated (to RCP45 scenario) first and second best carbon taxes and compares them to the necessary carbon taxes (first and second best) needed to limit the increase in the surface temperature to below the 2 °C target for the model where no carbon capture investment is utilized (no CC). In the bottom right panel, the left bar shows the necessary amount of CO_2 captured (in Giga tons of CO_2) under the calibrated model and whereas the right bar shows the necessary amount of CO_2 carbon capture needed to limit the increase in the surface temperature to below the 2 °C target in the special case that no carbon taxes are levied.

need to exceed 25.45 $GtCO_2$/year by mid-century, more than seven-fold increase from its baseline value of 3.77 $GtCO_2$/year required under the RCP4.5 scenario (Table 3). This sharp increase arises for two reasons. First, with zero carbon taxation, dirty capital investment remains at its current high level, maintaining elevated emissions. Second, in our model, the emission intensity of dirty capital η_d remains constant, so emissions can only be offset through removal rather than reduced at the source. At a cost of \$100–250 per ton of CO_2,[36] deploying 24 $GtCO_2$/year of carbon capture would cost 2.4–6 trillion USD per year, or approximately 2.82–7.06% of 2020 global GDP. This would constitute a significant proportion of overall government investment expenditures, which typically range around 16–18% of GDP.[37]

Moreover, the carbon-capture-only strategy faces a fundamental physical constraint. At a sustained rate of 24 $GtCO_2$/year, the prudent planetary storage limit of 1,460 $GtCO_2$[38] would be completely exhausted within just 60 years, i.e. by approximately 2080, rendering this approach physically unsustainable over longer horizons. Therefore, relying solely on either a carbon tax or carbon capture is suboptimal due to prohibitive economic costs (taxes high enough to induce rapid divestment) or binding physical constraints (exhaustion of geologic storage capacity). These findings underscore the necessity of the diversified policy portfolio analyzed in this paper, where moderate carbon taxes, clean subsidies, adaptation investments, and carbon capture work in concert to achieve climate stabilization at reasonable cost while respecting physical and economic constraints.

8 Conclusion

This paper re-evaluates the optimal design of climate policy in a world characterized by high damages, persistent carbon stocks, and transition frictions. By integrating mitigation, adaptation, and carbon removal into a unified general equilibrium framework, we demonstrate that the conventional reliance on carbon pricing is insufficient. The legacy stock of atmospheric CO_2 renders the flow-only approach of taxation economically inefficient and politically untenable.

Our analysis yields three robust conclusions. First, carbon dioxide removal is a prerequisite for achieving Paris Agreement targets, not a residual safety net. In our baseline calibration, satisfying the 2 °C constraint requires net carbon removal to scale from 0.04 to 3.7 $GtCO_2$/year

[36] See for example (National Academies of Sciences, Engineering, and Medicine, 2019; Smith et al., 2016) for an estimate of the costs of negative emission technologies.
[37] Based on World Bank data; see World Bank.
[38] See (Gidden et al., 2025).

by 2050, a ninety-fold increase. Meeting this target through taxation alone would require carbon prices reaching approximately \$474/tCO_2, which would violate the dirty-investment viability condition and imply large-scale divestment and scrapping. Under the implementability constraint, the required second-best carbon price rises from its baseline value to about \$214/tCO_2.

Second, the optimal climate policy is a portfolio defined by strong complementarities. The interaction between instruments generates welfare gains that exceed the sum of their individual parts. Adaptation preserves the economic surplus needed to fund the energy transition, while clean capital subsidies accelerate the reallocation of productive resources, lowering the necessary magnitude of the carbon tax. The optimal path exhibits distinct temporal dynamics: carbon pricing and subsidies dominate the near term to bend the emissions curve, while CDR scales aggressively in the mid-2030s to manage the atmospheric stock.

Third, welfare outcomes are highly sensitive to adaptation effectiveness. A 25% reduction in adaptation effectiveness increases expected damages by 37%, underscoring that resilience investment acts as critical insurance against tail risks. The physical limits of geological storage, estimated at 1,460 GtCO_2, imply that while CDR is essential, it cannot indefinitely substitute for abatement.

Our findings have direct implications for international climate architecture. If the social cost of carbon required to stabilize temperatures solely via pricing reaches approximately \$474/tCO_2 in our calibration, purely price-based agreements will fail due to participation constraints. Policy coordination must broaden to include technology transfer for negative emissions and resilience infrastructure. While our framework abstracts from geopolitical heterogeneity, the central message remains clear: solving the climate challenge requires moving beyond the pricing of externalities to the active management of the carbon stock and economic resilience.

A Proofs

A.1 Derivation of HJB equation

The optimal control problem of the government is a function of six state variables. Let $\mathbf{Q}_t = \big(X_t,\ \nu_{c,t},\ G_{s,t},\ E_{S,t},\ T_{S,t},\ T_{DO,t}\big)$. Thus, applying the infinitesimal generator $\mathcal{A}$ to $V(\mathbf{Q}_t)$ gives

$$\mathcal{A}V(\mathbf{Q}_t) = \underbrace{\sum_{i=1}^{n_{\mathbf{Q}}} \mu_i(\mathbf{Q}_t)\,\frac{\partial V}{\partial q_i}(\mathbf{Q}_t) + \frac{1}{2}\sum_{i=1}^{n_{\mathbf{Q}}}\sum_{j=1}^{n_{\mathbf{Q}}} \Sigma_{ij}(\mathbf{Q}_t)\,\frac{\partial^2 V}{\partial q_i\,\partial q_j}(\mathbf{Q}_t)}_{\text{(diffusion part)}}$$
$$+ \underbrace{\bar{\lambda}_X\,\frac{T_{S,t}}{T_{S,0}}\Big[V\big(\mathbf{Q}_t + \Delta_X(\mathbf{Q}_t)\big) - V(\mathbf{Q}_t)\Big]}_{\text{(jump in } X)} + \underbrace{\bar{\lambda}_{E_S}\,\frac{E_{S,t}}{E_{S,0}}\Big[V\big(\mathbf{Q}_t + \Delta_{E_S}(\mathbf{Q}_t)\big)\ -\ V(\mathbf{Q}_t)\Big]}_{\text{(jump in } E_S)} \tag{A.1}$$

where $\mu_i(\mathbf{Q}_t)$ is the drift rate of the i-th state variable, $\Sigma_{ij}(\mathbf{Q}_t)$ is the (i,j)-th entry of the diffusion (covariance) matrix. The first summation term $\sum_{i=1}^{n_{\mathbf{Q}}} \mu_i(\mathbf{Q}_t)\,\frac{\partial V}{\partial q_i}$ denotes the drift effect on V. The second summation term $\frac{1}{2}\sum_{i=1}^{n_{\mathbf{Q}}}\sum_{j=1}^{n_{\mathbf{Q}}} \Sigma_{ij}(\mathbf{Q}_t)\,\frac{\partial^2 V}{\partial q_i\,\partial q_j}$ is the diffusion effect on V. We denote the change in the state vector for this jump by $\Delta_X(\mathbf{Q}_t)$. The jump intensity for X_t is $\bar{\lambda}_X\,\frac{T_{S,t}}{T_{S,0}}$, i.e. upon a jump, X_t transitions from X_{t-} to $X_{t-} + X_{t-}\big(e^{-Z_X} - 1\big)$. Likewise, the jump intensity for $E_{S,t}$ is defined as $\bar{\lambda}_{E_S}\,\frac{E_{S,t}}{E_{S,0}}$. We denote the change in the state vector for this jump by $\Delta_{E_S}(\mathbf{Q}_t)$. As before, upon a jump, $E_{S,t}$ transitions from $E_{S,t-}$ to $E_{S,t-} + \zeta_E\, Z_{FB}$. Next, the HJB equation then is

$$0 = \sup_{C_t, I_t, G_t} f(C_t, V_t) + \mathcal{A}V(\mathbf{Q}_t) \tag{A.2}$$

where $\mathcal{A}V(\mathbf{Q}_t)$ is given in Equation (A.1). Since this is a closed economy with one representative household, in equilibrium aggregate wealth X_t must be equal to aggregate capital, i.e. $X_t = K_t$ which implies that aggregate output is given by $Y_t = AK_t$, i.e. a standard "AK" production function. Then, focusing on the special case with no R&D subsidy $G_{s,t} = 0$ and no feedback effects $\lambda_{FB} = 0$, differentiating the above expression with respect to consumption C_t, the investment policies $I_{c,t}$, $I_{d,t}$ and the government policies $G_{a,t}$ and $G_{cc,t}$, we obtain the following equations

$$\text{Consumption:} \quad (1-\gamma)V\beta/C_t - V_X = 0 \tag{A.3}$$

$$\text{Clean Capital Investment:} \quad \frac{\nu_{c,t} + w_{\nu_{c,t}}}{X_t} - \frac{\delta_c}{X_t} - \frac{\varphi_c I_{c,t}}{X_t^2} = 0 \tag{A.4}$$

$$\text{Dirty Capital Investment:} \quad \frac{\nu_d + w_{\nu_{d,t}}}{X_t} - \frac{\delta_d}{X_t} - \frac{\varphi_d I_{d,t}}{X_t^2} = 0 \tag{A.5}$$

$$\text{Adaptation Investment:} \quad -\frac{\nu_a}{2} - \sqrt{\frac{1}{\Lambda_t}\frac{G_{a,t}}{X_t}} = 0 \tag{A.6}$$

$$\text{Carbon Capture Investment:} \quad -\frac{\nu_{cc}\phi_{E_S}}{2} - (1-\gamma)\sqrt{\frac{1}{E_{S,t} - \bar{E}_T}\frac{G_{cc,t}}{X_t}} = 0 \tag{A.7}$$

First, substituting the explicit form of the value function guess into the first order condition of consumption yields the expressions in (33). Next, using the aggregate resource constraint $Y_t = AX_t = AK_t = C_t + I_t + G_t$, where $I_t = \sum_{j\in\{c,d,RD\}} I_{j,t}$ and $G_t = \sum_{j\in\{a,cc,r\}} G_{j,t}$, we solve this resource constraint for the residual government spending $G_{r,t}$, which, after substituting all the optimal policies from above shows that $G_{r,t}$ is linear in wealth X_t. Thus, this implies that all the policies are scaled by $1/X_t$, and thus, multiplying the HJB through by wealth X_t shows that it is independent of wealth, i.e.

$$
\begin{aligned}
0 = {} & \frac{\phi_{ES}^2\, FF^2}{2(\gamma-1)^2\,\varphi_d} + \frac{\nu_d^2\,\omega_s^2\, FF^2}{2\varphi_c} - \frac{\nu_d\,\phi_{ES}\, FF^2}{(\gamma-1)\,\varphi_d} + \frac{K_{dt}\,\phi_{ES}^2\,\phi_{Kd}\, FF^2}{2(\gamma-1)^3\,\varphi_d} - \frac{K_{dt}\,\nu_d\,\phi_{ES}\,\phi_{Kd}\, FF^2}{(\gamma-1)^2\,\varphi_d} \\
& + \frac{K_{dt}\,\nu_d^2\,\phi_{Kd}\, FF^2}{2(\gamma-1)\,\varphi_d} + \frac{\nu_d^2\, FF^2}{2\varphi_d} - \frac{\nu_{ct}\,\nu_d\,\omega_s^2\, FF}{\varphi_c} + \frac{\delta_d\,\phi_{ES}\, FF}{(\gamma-1)\,\varphi_d} - \frac{\phi_{ES}\, FF}{(\gamma-1)\,\varphi_d} \\
& + \frac{K_{dt}\,\delta_d\,\phi_{ES}\,\phi_{Kd}\, FF}{(\gamma-1)^2\,\varphi_d} - \frac{K_{dt}\,\delta_d\,\nu_d\,\phi_{Kd}\, FF}{(\gamma-1)\,\varphi_d} - \frac{\delta_c\,\nu_d\,\omega_s\, FF}{\varphi_c} + \frac{\nu_{ct}\,\nu_d\,\omega_s\, FF}{\varphi_c} + \frac{\nu_d\,\omega_s\, FF}{\varphi_c} \\
& - \frac{\delta_d\,\nu_d\, FF}{\varphi_d} + \frac{\nu_d\, FF}{\varphi_d} + \frac{E_{S,t}\,\epsilon_{cc}^2\,\phi_{ES}^2}{2(\gamma-1)^2} + \frac{\nu_{ct}^2\,\omega_s^2}{2\varphi_c} - A - \beta + \frac{\mathbb{E}\big(e^{-Z_K}-1\big)\, T_{S,t}\,\nu_a^2\,\lambda_K}{2\, T_{S,0}} \\
& + \frac{T_{S,t}\,\lambda_K \mathbb{E}\big(1-e^{Z_K(\gamma-1)}\big)}{(\gamma-1)\, T_{S,0}} + \frac{\theta_{CF}\,\kappa_{CF}\,\phi_{CFL}}{\gamma-1} - \frac{CF\,\kappa_{CF}\,\phi_{CFL}}{\gamma-1} - \frac{T_{S,t}\,\kappa_{CF}\,\upsilon_{TS}\,\phi_{CFL}}{\gamma-1} \\
& - \frac{E_{S,t}\,\delta_{ES}\,\phi_{ES}}{\gamma-1} - \frac{CF\,\xi_{CFL}\,\phi_{ES}}{\gamma-1} + \frac{K_{dt}\,\delta_d^2\,\phi_{Kd}}{2(\gamma-1)\,\varphi_d} - \frac{T_{DO,t}\,\kappa_{TOD}\,\phi_{TDO}}{\gamma-1} + \frac{T_{S,t}\,\kappa_{TOD}\,\phi_{TDO}}{\gamma-1} \\
& + \frac{E_{S,t}\,\alpha_{ES}\,\phi_{TS}}{\gamma-1} + \frac{T_{DO,t}\,\alpha_{TDO}\,\phi_{TS}}{\gamma-1} - \frac{T_{S,t}\,\alpha_{TDO}\,\phi_{TS}}{\gamma-1} - \frac{T_{S,t}\,\alpha_{TS}\,\phi_{TS}}{\gamma-1} \\
& + \frac{\bar{I}_{RD}\,\epsilon_{RD}\,\kappa_{\nu_c}\,\phi_{\nu_c}}{\gamma-1} + \frac{\theta_{\nu_c}\,\kappa_{\nu_c}\,\phi_{\nu_c}}{\gamma-1} - \frac{\kappa_{\nu_c}\,\nu_c\,\phi_{\nu_c}}{\gamma-1} - \frac{\kappa_{\nu_c}\,\nu_c^2\,\phi_{\nu_c^2}}{\gamma-1} \\
& + \frac{\bar{I}_{RD}\,\epsilon_{RD}\,\kappa_{\nu_c}\,\nu_c\,\phi_{\nu_c^2}}{\gamma-1} + \frac{\theta_{\nu_c}\,\kappa_{\nu_c}\,\nu_c\,\phi_{\nu_c^2}}{\gamma-1} - \frac{\nu_{ct}^2\,\omega_s}{\varphi_c} + \frac{\delta_c\,\nu_{ct}\,\omega_s}{\varphi_c} - \frac{\nu_{ct}\,\omega_s}{\varphi_c} \\
& - \frac{\beta}{\gamma-1}\Big(\frac{1}{2}\phi_{\nu_c^2}\,\nu_{ct}^2 + \phi_{\nu_c}\,\nu_{ct} + \phi_0 + CF\,\phi_{CFL} + E_{S,t}\,\phi_{ES} + K_{dt}\,\phi_{Kd} + T_{DO,t}\,\phi_{TDO} + T_{S,t}\,\phi_{TS}\Big) \\
& + \beta\log\beta + \frac{\zeta_{ET}\,\phi_{ES}\,\bar{e}_N}{\gamma-1} - \frac{\epsilon_{cc}^2\,\phi_{ES}^2\,\overline{E}_N}{2(\gamma-1)^2} + \frac{K_{dt}\,\zeta_{ET}\,\eta_d\,\phi_{ES}\,u(\tau_d)}{\gamma-1} + \frac{\delta_c^2}{2\varphi_c} + \frac{\nu_{ct}^2}{2\varphi_c} \\
& - \frac{\delta_c}{\varphi_c} - \frac{\delta_c\,\nu_{ct}}{\varphi_c} + \frac{\nu_{ct}}{\varphi_c} + \frac{\delta_d^2}{2\varphi_d} - \frac{\delta_d}{\varphi_d}\,. \qquad \text{(A.8)}
\end{aligned}
$$

Next, we collect first all the expressions in the HJB equation above, which multiply the state variables $v_{c,t}$, $v_{c,t}^2$, $K_{d,t}$, $E_{S,t}$, CF_t, $T_{S,t}$ and $T_{DO,t}$, set the resulting expression to zero and then solve for the value function coefficients $\phi_{\nu_c,\ell}, \phi_{K_d}, \phi_{\nu_c,q}, \phi_{E_S}, \phi_{CF}, \phi_{T_S}, \phi_{T_{DO}}$ which yields the following non-linear system of equations

$$\begin{aligned}
\phi_{\nu_c^2} &= \frac{(\gamma-1)(1-\omega_s)^2}{\beta+2\kappa_{\nu_c}},\ \phi_{CF} = -\frac{\xi_{CF}\phi_{E_S}}{(\beta+\kappa_{CF})},\ \phi_{T_{DO}} = \frac{\alpha_{T_{DO}}\phi_{T_S}}{(\beta+\kappa_{T_{DO}})},\\
\phi_{\nu_c} &= \frac{\kappa_{\nu_c}(\theta_{\nu_c}+\bar{I}_{RD})\varphi_c\phi_{\nu_c^2}+(\gamma-1)(1-\omega_s)(1-\delta_c+\omega_s\nu_d FF)}{(\beta+\kappa_{\nu_c})\varphi_c},\\
\phi_{K_d} &= \frac{2(\gamma-1)^2\zeta_E\eta_d\varphi_d\phi_{E_S}u(\tau_d)}{(\gamma-1)^2\left[(\nu_d FF-\delta_d)^2-2\beta\varphi_d\right]+2FF(\gamma-1)(\nu_d FF-\delta_d)\phi_{E_S}+FF^2\phi_{E_S}^2},\\
\phi_{E_S} &= \frac{(\beta+\delta_{E_S})(\gamma-1)\pm\sqrt{(1-\gamma)\left[(1-\gamma)(\beta+\delta_{E_S})^2+2\alpha_{E_S}\phi_{T_S}\epsilon_{cc}^2\right]}}{\epsilon_{cc}^2}\\
\phi_{T_S} &= \frac{(\beta+\kappa_{T_{DO}})\left(\frac{\kappa_{CF}\xi_{CF}\upsilon_{T_S}\phi_{\mathrm{ES}}}{\beta+\kappa_{CF}}-\frac{\lambda_X\left((1-\gamma)\frac{\nu_a^2}{2}(1+\beta_{Z_X})^{-\alpha_{Z_X}}+(1+(1-\gamma)\beta_{Z_X})^{-\alpha_{Z_X}}\right)}{T_{S,0}}\right)}{(\alpha_{T_{DO}}\beta+(\alpha_{T_S}+\beta)(\beta+\kappa_{T_{OD}}))}
\end{aligned}$$

where we have used the fact that $Z_X \sim \mathrm{Gamma}(\alpha_X, \beta_X)$ in order to obtain fully closed-form expressions of the jump size integrals. Finally, after all the state-dependent term have been removed, the remaining constant terms are also set to zero and we then solve for the value function term ϕ_0 explicitly. Since this term is very complicated and lengthy, we omit it here.

A.2 From the Optimal Capital Tax to a Carbon Price in USD/tCO$_2$

We start from the scaled social cost of carbon (SCC) which we define as follows

$$\widetilde{SCC} := \frac{SCC_t}{X_t} = \frac{\phi_{E_S}}{1-\gamma}, \qquad SCC_t := -\frac{\partial V/\partial E_{S,t}}{\partial V/\partial X_t}, \tag{A.9}$$

which is constant in our HJB solution and has units 1/ppm (since $E_{S,t}$ is measured in ppm and $-\phi_{E_S}E_{S,t}$ enters the value function). Define baseline ratios

$$\chi_d := \frac{E_{T,0}}{FF_0}, \qquad \eta_d := \frac{E_{T,0}}{K_{d,0}}, \tag{A.10}$$

where $E_{T,0}$ is in GtCO$_2$/year, FF_0 in EJ/year, and $K_{d,0}$ in trillion USD. Hence $[\chi_d]$ = GtCO$_2$/EJ and $[\eta_d]$ = GtCO$_2$/(trn USD · year). The ppm–to–Gt conversion factor satisfies $[\zeta_E]$ = ppm/GtCO$_2$. Next, recall that the dirty-capital drift for the household is given by

$$\mu^H_{K_{d,t}} = (1-\tau_{d,t})\,\nu_d\,FF_t\,\frac{I_{d,t}}{X_t} - \left[\frac{\varphi_d}{2}\left(\frac{I_{d,t}}{X_t}\right)^2 + \delta_d\,\frac{I_{d,t}}{X_t}\right], \tag{A.11}$$

where ν_d converts fossil-energy input (EJ) into a contribution to the expected return (units 1/(EJ · year)). Let $SCC_t^{EJ} := SCC_t \cdot \zeta_E\chi_d$ denote the SCC per unit of fossil energy (per EJ per year). Internalizing a fraction $\varsigma \in (0,1]$ of the first-best wedge implies

$$\tau^*_{d,t} = \varsigma\, \frac{SCC^{EJ}_t/X_t}{\nu_d} = \varsigma\, \frac{\zeta_E\, \chi_d}{\nu_d}\, \widetilde{SCC}, \tag{A.12}$$

so $\tau^*_{d,t}$ is a pure number in $(0,1)$. Next, let $u_t(\tau_d) \in (0,1]$ be the (monotone) utilization factor that maps the tax into the operating intensity of dirty capital.[39] Emissions from dirty capital are

$$E^{\text{dirty}}_t = \eta_d\, u_t(\tau_d)\, K_{d,t} \qquad [\text{GtCO}_2/\text{year}]. \tag{A.13}$$

For the tax base, map the return wedge $\tau_{d,t}$ to an ad-valorem rate on the *gross* dirty services flow per unit of capital $r^g_d := \nu_d\, FF_t \quad [1/\text{year}]$ so tax revenue is

$$\text{Tax}_t = \tau_{d,t}\, r^g_d\, K_{d,t} \qquad [\text{trn USD/year}]. \tag{A.14}$$

The model-implied carbon price equals tax revenue per unit of emissions:

$$p^{CO_2}_t := \frac{\text{Tax}_t}{E^{\text{dirty}}_t} = \frac{\tau_{d,t}\, r^g_d\, K_{d,t}}{\eta_d\, u_t(\tau_d)\, K_{d,t}} = \tau_{d,t}\, \frac{r^g_d}{\eta_d\, u_t(\tau_d)} \qquad \big[\text{trn USD/Gt}\big]. \tag{A.15}$$

Using $1\,\text{trn USD/Gt} = 10^3\,\text{USD/t}$ gives the following expression[40]

$$p^{CO_2}_t\big[\text{USD/t}\big] = 10^3\ \tau_{d,t}\, \frac{\nu_d\, FF_t}{\eta_d\, u_t(\tau_d)}. \tag{A.16}$$

A.3 Sensitivity of value-function coefficients ϕ_{E_S} and ϕ_{T_S}

Proposition A.1 (Comparative statics of ϕ_{E_S} and ϕ_{T_S})**.** *Let ϕ_{E_S} and ϕ_{T_S} satisfy*

$$\phi_{E_S} = \frac{(\beta+\delta_{E_S})(\gamma-1)\ \pm\ \sqrt{\Delta}}{\nu^2_{cc}}, \qquad \Delta := (1-\gamma)\big[(1-\gamma)(\beta+\delta_{E_S})^2 + 2\alpha_{E_S}\phi_{T_S}\nu^2_{cc}\big], \tag{A.17}$$

$$\phi_{T_S} = \frac{(\beta+\kappa_{T_{DO}})\Big(\frac{\kappa_{CF}\xi_{CF}\upsilon_{T_S}}{\beta+\kappa_{CF}}\phi_{E_S} - \frac{\lambda_{Z_X}}{T_{S,0}}\Big(\frac{1-\gamma}{2}\nu^2_a(1+\beta_{Z_X})^{-\alpha_{Z_X}} + (1+(1-\gamma)\beta_{Z_X})^{-\alpha_{Z_X}}\Big)\Big)}{\alpha_{T_{DO}}\beta + (\alpha_{T_S}+\beta)(\beta+\kappa_{T_{DO}})}. \tag{A.18}$$

Choose the root such that $\phi_{E_S} > 0$ for $\gamma < 1$ and $\phi_{E_S} < 0$ for $\gamma > 1$, ensuring that the value function decreases in $E_{S,t}$ and $T_{S,t}$. Then:

---

[39] In the baseline implementation we use a smooth, one-parameter exponential:

$$u_t(\tau_d) = u_{\min} + (1-u_{\min})\exp\Big(-\kappa_u\, \frac{\tau_d}{r^n_d}\Big), \quad r^n_d := \max\{\nu_d FF_t - \delta_d,\, 10^{-6}\}.$$

Any monotone, bounded choice works in the derivations below.

[40] $r^g_d = \nu_d FF_t$ has units 1/year and η_d has units $\text{GtCO}_2/(\text{trn USD}\cdot\text{year})$, so r^g_d/η_d is trn USD/GtCO_2. Multiplying by 10^3 yields USD/tCO_2, as in (A.16).

1. ϕ_{E_S} and ϕ_{T_S} always move in the same direction:

$$\frac{\partial \phi_{T_S}}{\partial \phi_{E_S}} > 0, \qquad \frac{\partial \phi_{E_S}}{\partial \phi_{T_S}} = -\frac{(1-\gamma)\alpha_{E_S}}{\sqrt{\Delta}}.$$

2. For $\gamma < 1$:

- *Higher adaptation or carbon-capture effectiveness (ν_a, ν_{cc}) reduces ϕ_{E_S} and ϕ_{T_S}.*
- *Higher disaster intensity or persistence ($\lambda_{Z_X}, \alpha_{Z_X}, \beta_{Z_X}$) raises ϕ_{T_S} but lowers ϕ_{E_S}.*
- *A higher discount rate (β) or CO_2 depreciation (δ_{E_S}) lowers ϕ_{E_S} and ϕ_{T_S}; higher risk aversion (γ) increases ϕ_{E_S}.*

3. For $\gamma > 1$, the signs reverse for $\nu_a, \nu_{cc}, \gamma, \delta_{E_S}$, while the effects of $\lambda_{Z_X}, \alpha_{Z_X}, \beta_{Z_X}$ and β are ambiguous.

Proof. We proceed by differentiating (A.18) and (A.17) and analyzing the signs of each term. Since ϕ_{T_S} depends linearly on ϕ_{E_S} with positive slope, we have:

$$\frac{\partial \phi_{T_S}}{\partial \phi_{E_S}} > 0.$$

Differentiating ϕ_{E_S} with respect to ϕ_{T_S} (holding other parameters fixed), we find:

$$\frac{\partial \phi_{E_S}}{\partial \phi_{T_S}} = -\frac{(1-\gamma)\alpha_{E_S}}{\sqrt{\Delta}}.$$

Thus, ϕ_{E_S} and ϕ_{T_S} always move in the same direction, with the direction of $\partial \phi_{E_S}/\partial \phi_{T_S}$ depending on $(1-\gamma)$. Next, the adaptation effectiveness parameter ν_a only enters (A.18), not (A.17) directly. We compute:

$$\frac{\partial \phi_{T_S}}{\partial \nu_a} = -\frac{(\beta + \kappa_{T_{DO}}) \cdot \lambda_{Z_X}}{T_{S,0} D_T} \cdot (1-\gamma)\nu_a (1+\beta_{Z_X})^{-\alpha_{Z_X}},$$

with $D_T > 0$. Hence:

$$\frac{\partial \phi_{T_S}}{\partial \nu_a} < 0 \text{ if } \gamma < 1, \quad > 0 \text{ if } \gamma > 1.$$

Then, via the chain rule:

$$\frac{\partial \phi_{E_S}}{\partial \nu_a} = \frac{\partial \phi_{E_S}}{\partial \phi_{T_S}} \cdot \frac{\partial \phi_{T_S}}{\partial \nu_a}.$$

Since both factors are negative when $\gamma < 1$, this gives $\partial \phi_{E_S}/\partial \nu_a < 0$. Same logic applies for $\gamma > 1$, resulting again in $\partial \phi_{E_S}/\partial \nu_a < 0$. The carbon capture effectiveness parameter ν_{cc} appears directly in ϕ_{E_S}. Differentiating (A.17) yields:

$$\frac{\partial \phi_{E_S}}{\partial \nu_{cc}} = -\frac{2\phi_{E_S}}{\nu_{cc}} + \frac{2(1-\gamma)\alpha_{E_S}\phi_{T_S}}{\nu_{cc}\sqrt{\Delta}}.$$

For $\gamma < 1$, we have $\phi_{E_S} > 0$ and $\phi_{T_S} > 0$, so both terms are negative and $\partial\phi_{E_S}/\partial\nu_{cc} < 0$. For $\gamma > 1$, both terms are positive, so $\partial\phi_{E_S}/\partial\nu_{cc} > 0$. Next, the disaster parameters: $\lambda_{Z_X}, \alpha_{Z_X}, \beta_{Z_X}$ enter the damage term in ϕ_{T_S} positively. Therefore, it follows that

$$\frac{\partial\phi_{T_S}}{\partial p} > 0, \quad \Rightarrow \quad \frac{\partial\phi_{E_S}}{\partial p} = \frac{\partial\phi_{E_S}}{\partial\phi_{T_S}} \cdot \frac{\partial\phi_{T_S}}{\partial p} \text{ has opposite sign of } (1-\gamma).$$

Thus: - For $\gamma < 1$, $\partial\phi_{E_S}/\partial p < 0$, - For $\gamma > 1$, $\partial\phi_{E_S}/\partial p > 0$. The CO_2 depreciation: δ_{E_S} appears in both terms of ϕ_{E_S}. For the first term we get $\frac{\partial}{\partial\delta_{E_S}}\left[\frac{(\beta+\delta_{E_S})(\gamma-1)}{\nu_{cc}^2}\right] = \frac{\gamma-1}{\nu_{cc}^2}$ and for the second term we obtain $\frac{\partial}{\partial\delta_{E_S}}\left[\frac{\sqrt{\Delta}}{\nu_{cc}^2}\right] = \frac{(1-\gamma)^2(\beta+\delta_{E_S})}{\nu_{cc}^2\sqrt{\Delta}}$. Putting these results together we get

$$\frac{\partial\phi_{E_S}}{\partial\delta_{E_S}} = \frac{\gamma-1}{\nu_{cc}^2} - \frac{(1-\gamma)^2(\beta+\delta_{E_S})}{\nu_{cc}^2\sqrt{\Delta}}.$$

This expression is negative for $\gamma < 1$, and potentially ambiguous for $\gamma > 1$. But empirically, we find ϕ_{E_S} decreases in δ_{E_S}, consistent with $\partial\phi_{E_S}/\partial\delta_{E_S} < 0$ under both cases. Next, for the discount factor β we get

$$\frac{\partial\phi_{E_S}}{\partial\beta} = \frac{\gamma-1}{\nu_{cc}^2} - \frac{(1-\gamma)^2(\beta+\delta_{E_S})}{\nu_{cc}^2\sqrt{\Delta}}.$$

For $\gamma < 1$, both terms are negative, so ϕ_{E_S} decreases in β. Finally, regarding the risk aversion coefficient γ, we start by differentiating both the numerator and square root to obtain

$$\frac{\partial\phi_{E_S}}{\partial\gamma} = \frac{\beta+\delta_{E_S}}{\nu_{cc}^2} + \frac{(1-\gamma)(\beta+\delta_{E_S})^2 + \alpha_{E_S}\phi_{T_S}\nu_{cc}^2}{\nu_{cc}^2\sqrt{\Delta}}.$$

For $\gamma < 1$, all terms are positive, so ϕ_{E_S} increases in γ. For $\gamma > 1$, sign is ambiguous due to potential nonlinearity in Δ. □

A.4 Expected clean capital path and long run productivity

Using the optimal clean capital investment rule of the government $I_{c,t}^{G,*}/X_t = \frac{\nu_{c,t}+w_{\nu_{c,t}}-\delta_c}{\varphi_c}$ and plug it into the clean capital dynamics in Equations (8) and (11) we obtain,

$$\frac{dK_{c,t}^{*,G}}{K_{c,t}^{*,G}} = \frac{\left((1-\omega_s)\nu_{c,t} + \omega_s\nu_d FF - \delta_c\right)^2}{2\varphi_c}dt \tag{A.19}$$

where we have set the volatility term $\sigma_{K_c} = 0$, so that the diffusive term vanishes. Recall that clean productivity $\nu_{c,t}$ evolves as $d\nu_{c,t} = \left(\theta_{\nu_c} + \epsilon_{I_{RD}}I_{RD} - \delta_{\nu_c}\nu_{c,t}\right)dt + \sigma_{\nu_c}dB_t^{\nu_c}$. Since $I_{RD,t} = I_{RD}\cdot X_t$ and I_{RD} is a constant share of wealth devoted to R&D investment, then the expected path of $\nu_{c,t}$ is $\mathbb{E}[\nu_{c,t}] = \psi_{\nu_c}(t) = \theta_{RD} + e^{-\delta_{\nu_c}t}\left(\nu_{c,0} - \theta_{RD}\right)$ where we define $\theta_{RD} := \theta_{\nu_c} + \epsilon_{I_{RD}}I_{RD}$

and $\delta_{\nu_c} > 0$. The long-run growth rate of clean capital is controlled by the term

$$b(\omega_s) := \frac{1}{2\varphi_c}\left[\theta_{RD} - \omega_s(\theta_{RD} - \nu_d FF) - \delta_c\right], \quad \Rightarrow \quad \frac{\partial b}{\partial \omega_s} = \frac{1}{2\varphi_c}\left(\nu_d FF - \theta_{RD}\right).$$

This leads us to the following decision rule for ω_s: The optimal value of ω_s depends on the relative size of $\nu_d FF$ and θ_{RD}: If $\nu_d FF > \theta_{RD}$, then $\frac{\partial b}{\partial \omega_s} > 0 \Rightarrow$ increasing ω_s increases growth, which implies that the optimal policy is $\omega_s = 1$, i.e. rely entirely on direct subsidies to clean capital. Next if $\nu_d FF < \theta_{RD}$, then $\frac{\partial b}{\partial \omega_s} < 0 \Rightarrow$ increasing ω_s reduces growth and in this case it is optimal to set $\omega_s = 0$, i.e. rely entirely on R&D-driven productivity growth.[41] Thus, if R&D investment is able to increase long run clean capital productivity beyond the productivity of dirty capital, this determines whether the subsidy should compensate any initial productivity differential between the two capital types ($\omega_s = 1$) or whether it should be used at all ($\omega_s = 0$). However, in the short- to medium-run, a subsidy is needed to jump-start clean capital accumulation, even though its long-run growth path is lower compared to the case when only R&D investment is used.

A.5 Deriving the Expectation of Wealth $\mathbb{E}_t[X_T]$

We consider the stochastic differential equation governing the evolution of aggregate wealth X_t in our economy. The dynamics are given by

$$\frac{dX_t}{X_t} = \sum_{j\in\{c,d,r\}} \mu_{K_j,t}(\pi_{j,t}) - \beta - \frac{G_t}{X_t} - \frac{I_{RD,t}}{X_t} - f_a(G_{a,t})\Lambda(t) + (e^{-Z_X} - 1)dN_t^{\lambda_X}, \tag{A.20}$$

where $\mu_{K_j,t}(\pi_{j,t})$ is the net return on capital j (clean, dirty, residual), which depends on the investment share $\pi_{j,t} = \frac{I_{j,t}}{X_t}$, G_t is government expenditure, including adaptation, carbon capture, and public spending, $I_{RD,t}$ is R&D investment, $f_a(G_{a,t})\Lambda(t)$ is the damage mitigation cost from adaptation investments and $(e^{-Z_X} - 1)dN_t^{\lambda_X}$ is a Poisson jump process capturing climate disaster shocks. The optimal policy rules imply constant or time-varying investment shares

$$\pi_{c,t} = \frac{I_{c,t}}{X_t} = \frac{\nu_{c,t} + w_{\nu_c} - \delta_c}{\varphi_c}, \quad \pi_d = \frac{I_{d,t}}{X_t} = \frac{\nu_d FF - \delta_d}{\varphi_d},$$
$$\pi_r = \frac{I_{r,t}}{X_t} = \frac{\nu_r - \delta_r}{\varphi_r}, \quad \pi_{RD} = \frac{I_{RD,t}}{X_t} = \text{constant}.$$

The net returns on capital are given by

$$\mu_{K_c,t} = \nu_{c,t}\cdot\pi_{c,t} - \left(\frac{\varphi_c}{2}\pi_{c,t}^2 + \delta_c\pi_{c,t}\right),\ \mu_{K_d} = \nu_d FF\cdot\pi_d - \left(\frac{\varphi_d}{2}\omega_d^2 + \delta_d\cdot\pi_d\right),$$

[41] If $\nu_d FF = \theta_{RD}$ then the growth rate is independent of ω_s.

where ω_d is the dirty capital-to-wealth ratio, assumed to be approximately stationary. Next, the total government expenditure relative to wealth is given by

$$\frac{G_t}{X_t} = \pi_{a,t} + \pi_{cc,t} + \pi_s + \pi_o(t) = A - \beta - \pi_I,$$

where $\pi_{a,t}, \pi_{cc,t}$ are time-varying shares for adaptation and carbon capture, π_s is the share of fixed public spending, $\pi_o(t)$ is residual government spending, pinned down via the resource constraint and $\pi_I := \pi_c + \pi_d$ is the total capital investment share. Using the above components, we rewrite equation (A.20) as

$$\frac{dX_t}{X_t} = \mu_{K_c,t} + \mu_{K_d} + \mu_{K_r} - A + \pi_I - \pi_{RD} - f_a(G_{a,t})\Lambda(t) + (e^{-Z_X} - 1)dN_t^{\lambda_X}. \tag{A.21}$$

We now define

$$\begin{aligned} f_{\text{tv}}(t) &:= \mu_{K_c,t} - A - f_a(G_{a,t})\Lambda(t), \quad \text{(time-varying part)} \\ \bar{f} &:= \mu_{K_d} + \mu_{K_r} + \pi_I - \pi_{RD}, \quad \text{(constant part)} \end{aligned}$$

so that the dynamics become

$$\frac{dX_t}{X_t} = \left(f_{\text{tv}}(t) + \bar{f}\right) dt + (e^{-Z_X} - 1)dN_t^{\lambda_X}. \tag{A.22}$$

Then, we compute the expectation of wealth using

$$\mathbb{E}_t[X_T] = X_t \cdot \mathbb{E}_t\left[\exp\left(\int_t^T \left(f_{\text{tv}}(s) + \bar{f}\right) ds + \int_t^T \log(e^{-Z_X})dN_s^{\lambda_X}\right)\right] \tag{A.23}$$

By the properties of compound Poisson processes, we have

$$\mathbb{E}_t\left[\exp\left(\int_t^T \log(e^{-Z_X})dN_s^{\lambda_X}\right)\right] = \exp\left(-\int_t^T \lambda_X(s)\left(1 - \mathbb{E}[e^{-Z_X}]\right) ds\right) \tag{A.24}$$

Assuming $Z_X \sim \text{Gamma}(\alpha_X, \beta_X)$ (shape-scale), the Laplace transform is $\mathbb{E}[e^{-Z_X}] = \left(1 + \frac{1}{\beta_X}\right)^{-\alpha_X}$. Putting all the components together, we obtain

$$\mathbb{E}_t[X_T] = X_t \cdot \exp\left(\int_t^T \left[f_{\text{tv}}(s) + \bar{f} - \lambda_X(s)\left(1 - \left(1 + \frac{1}{\beta_X}\right)^{-\alpha_X}\right)\right] ds\right) \tag{A.25}$$

This expression will be used to compute model-implied output through the relation $Y_t = AX_t$, which is critical for calibration to observed GDP levels.

B Model Calibration: Details

We calibrate the model by minimizing a composite loss function that jointly disciplines the climate block and the economic block. On the climate side, we target the CO_2 concentration and surface temperature paths under both RCP4.5 (policy scenario) and RCP8.5 (business-as-usual). For each scenario, we compute the squared relative deviation between the model-implied CO_2 and temperature series and their RCP counterparts, but we place more weight on the first three decades of the transition. Concretely, for $t \leqslant T_E = 30$ we upweight the errors by a factor $w_{\text{early}} = 2$ (and $1.5 \times w_{\text{early}}$ for temperatures), reflecting that the near- and medium-term evolution of climate variables is both better measured and more relevant for the economic decisions we study. We also give substantially more weight to the RCP4.5 targets than to RCP8.5: the latter enters the objective with a factor $w_{\text{BAU}} = 0.1$, so that RCP8.5 is used primarily as a consistency check that the "all-policies-off" configuration reproduces a reasonable business-as-usual path, but does not dominate the calibration.

On the economic side, we match a vector of key 2020 and 2050 aggregates (GDP, clean and dirty investment, clean output share, adaptation spending, economic damages, and carbon capture volumes, together with the 2050 carbon tax) to their empirical or constructed targets. For each moment, we work with relative deviations of model from target, and we aggregate them using a Huber loss: small deviations are penalized quadratically, while deviations beyond a threshold $\delta_{\text{Error}} = 0.05$ are penalized linearly. This increases the robustness of our calibration against a few hard-to-match moments and prevents the objective from being dominated by outliers or slight mismeasurement in constructed "data" for 2050. Moment-specific importance weights allow us to emphasize some targets (e.g., GDP, dirty investment, damages) more than others, and the entire economic block enters the objective scaled by 2/3 relative to the climate-error component. The resulting total loss is minimized over the parameter vector using a genetic algorithm with multiple random initializations and parallel computation, subject to economic and physical feasibility constraints (e.g., positive net returns on dirty capital, well-behaved value-function coefficients). Only parameter draws that satisfy these constraints and achieve a sufficiently low objective value are retained as admissible calibrations.

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